ArchNet IJAR
International Journal of Architectural Research

BREAKING SYMMETRIES AND EMERGING SCALING URBAN STRUCTURES
A Morphological Tale of 3 Cities: Paris, New York and Barcelona

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Abstract
The challenge of a science of cities is to understand the links between urban morphogenesis, efficiency and resilience. Mathematical regularities emerge in resilient cities, coming from the scale-free properties of complex systems that present the same level of complexity across their different scales. They take the form of inverse power laws that are the « signature » of complexity. In living cities, these mathematical regularities derive from historical layering over millennia (Paris) or from intense market forces (New York). In complex, living and resilient cities, the distribution of elements and connections does not obey Gaussian laws but scale-free inverse power laws. Understanding the universality of this structure which also characterizes natural phenomena and living systems, and which has been violated by modernist city planning, would allow planning more efficient and resilient cities. The paper shows how initial breaks of symmetry fostered the emergence of scale-free structures in Paris and New York, with long-range time correlations, and how a break of symmetry in the spatial layout created a highly differentiated socio-economic structure in Barcelona.

Keywords: urban morphology; symmetries; scaling

THE CHALLENGE OF A NEW SCIENCE OF CITIES

The challenge of a new science of cities is to understand the links between urban morphogenesis, efficiency and resilience. It is also to understand the relationships between self-organization and planning. The large number and diversity of agents operating simultaneously in a city suggest that cities are a multi-fractal emergent phenomenon. On the other hand, planning plays an important role in the city, leaving long standing traces, and could be thought of as an external perturbation, as if it were foreign to the self-organized development of a city. Fractal geometry and complex systems theories reveal mathematical regularities maintained through the seemingly chaotic process of urban change. Fractal geometry has contributed to climate modeling, to study turbulent flows, to analyze brain waves and seismic movements as well as to understand the distribution of galaxies. It has also transformed finance by revealing a hidden complex order in the seemingly chaotic fluctuation of prices. It should transform the study and planning of cities. Historical cities display a multi-fractal structure layering and interlocking different fractal structures belonging to different morphological periods; the fractal and scaling parameters display high local variations (singularities) which are organized in different fractal sets (isohölder) described by a spectrum of fractal dimensions (Haussdorf spectrum).

In other words, are averages (average density, average GDP, average energy intensity or GHG emissions) meaningful in urban studies? In a Gaussian world they are meaningful because 68% of the values are at one standard deviation from the average. Quite the opposite, a Paretian world is extremely unequal: a few extremely high values are juxtaposed to a “long tail” of very low values.

TWO DIFFERENT KINDS OF SYMMETRY: MODERNIST PLANNING TRANSLATION SYMMETRY VERSUS HISTORICAL SCALING SYMMETRY

We know that symmetry plays a fundamental role in physical phenomena. Symmetries are certain properties of laws of physics that are conserved when a system undergoes a given geometric transformation. Equations in physics are, for example, invariant by translation in space and time. When we look for the most fundamental nature of physical interactions, we always find properties of symmetry. Emmy Noether was the first to discover that fundamental symmetries are at the origin of physical laws, such as energy conservation and the constant of motion. From this perspective, fractal geometry corresponds to a form of symmetry that is dilatation symmetry or scale invariance. It is found in countless natural phenomena and in living organisms whose evolution favored fractal structures because of the efficiency and resilience they offer.

Le Corbusier's modernism relies only on translation symmetries, repeating the same oversized objects in a highly simplified space with only one scale.

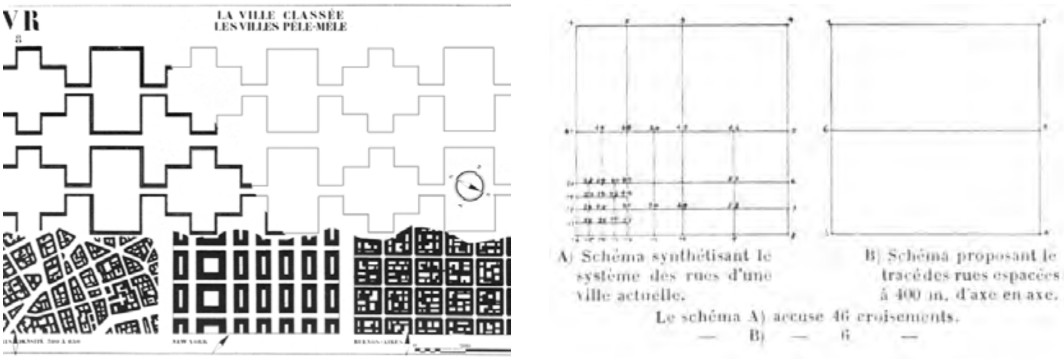

Figure 1: Drawings from Le Corbusier's Urbanisme (Corbusier 1922): la ville classée et la ville pêle-mêle. Le Corbusier proposed to remove all the small and intermediary scales of the urban structure (Source: Authors).

Le Corbusier removes from the Radiant City all the smaller and intermediary scales of the historical city to replace them by a giant scale duplicated by translational symmetry. Le Corbusier compares the sizing and inner complexity of Paris, New York and Buenos Aires urban blocks with the highly simplified and repetitive type of the Radiant City. He also shows an abstracted version of the scaling of the historical street patterns with 46 intersections on a square 400 meters side, that is 163 intersections/km2 taking into account edges effect, i.e. the density of intersections of Paris and Manhattan. Le Corbusier's scheme of historical cities leads to a linear density of 20km/km2 (22km/km2 for present Paris intra-muros). Le Corbusier scheme is clearly an attack on the connective properties of the historical urban fabric in order to replace them by the car-oriented new paradigm: his proposed superblock scheme with only 6 intersections, leading to 18 intersections/km2 and a the linear density of roads of 8km/km2. Contemporary Chinese planners have reduced further this linear density to 4 to 3km/km2.

Quite the opposite, from their multicellular growth over long periods of time historical cities have developed a scaling symmetry. In scale-free systems, inverse power laws act as a link between scales: the frequency of an element of size x is proportional to the inverse of its size at a scaling exponent m characteristic of the scaling properties of the system. There are few big elements, a medium number of medium-scale elements and a very large number (a "long tail") of small-scale elements. The relative frequency of each type is determined by the scaling parameter of the inverse power law.

In the Nolli-Piranesi plan of Roma of 1748, the hollow connective spaces are streets, outside squares but also interior of palace courtyards and churches. Studies have calculated the fractal scaling parameters of these types of urban fabric both for street patterns and for built forms (Frankhauser 1994).

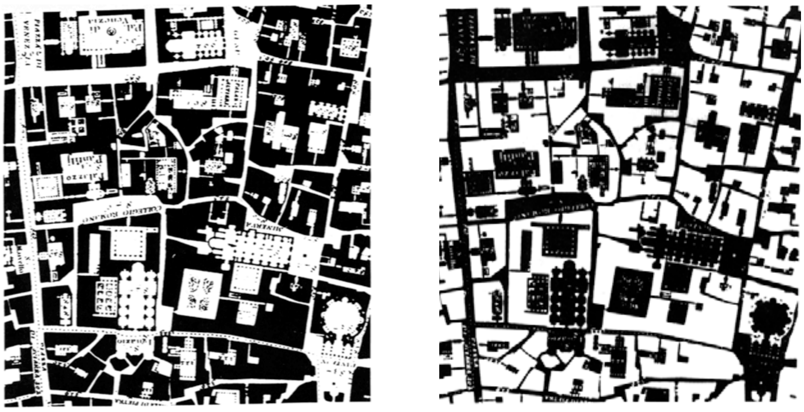

Figure 2 : Detail of Nolli-Piranesei map of Roma (Source: Tice and Steiner 2013).

WHERE DOES URBAN SCALING HIERARCHY COME FROM?

Like all living organisms, cities are evolutionary open systems. They are shaped and constantly transformed by social and political struggle, and by market forces. Their complexity is an emergent phenomenon based on equilibrium between bottom-up interactions of a great multiplicity of competing agents and top-down planning interventions. Too much competition may lead to anarchy while too much planning may constrain the system evolution. Urban systems constantly exchange people, ideas, goods, energy and matter with their environment. Open systems like cities cannot be described using classical thermodynamics, as developed in the 19th century by Boltzmann. To understand them, we have to turn to the more recent work of Ilya Prigogine on the theory of dissipative structures, far-from-equilibrium thermodynamics, and self-organizing systems where breaks of symmetry create scale-free patterns.

Breaking symmetries and the emergence of scale-free structures

Emergence requires first a fine grain structure, a high number of connections and a vast combinatorial space of configurations. Second, emergence requires breaks of symmetry in streets metric and topological properties, in division of land into plots. Avenues in Manhattan for example are 13 times more continuous and connective than streets not only because avenues are longer and wider than streets, but above all because they connect 155 streets while streets connect only 11 avenues. Blocks sizes in Manhattan are not square but elongated, which reinforces the previous break of symmetry.

Why is scaling hierarchy a key factor of urban sustainability and resilience?

Historical cities, like Paris with its 2000 years long history, were slowly transformed by incremental phenomena of destruction and reconstruction of the urban fabric. Structures that were not resilient enough were eliminated. And so historical cities have come down to us with extraordinary capacities of efficiency and resilience. In a process of spontaneous self-organization to adapt their forms to fluctuations in their environment, historical cities acquired the capacity to absorb fluctuations by reinforcing their structure and order, and becoming more complex.

Resilience may be defined as the ability of a system to evolve while keeping embedded in its structure the memory of its previous states. Transformation increases the number of scales without destroying the previous existing scales. It reinforces the scaling structure by enlarging it toward higher scales (Haussmann in Paris), by diversifying an original highly symmetrical state into a scaling structure (New York evolution under market forces), by intensifying it towards smaller scales (Tokyo or Kyoto plot fragmentation). In all cases the result of living evolutionary processes is reinforcing scaling structures. Quite the opposite, modernist top down planned cities that have no scaling structure like Le Corbusier Radiant City or like China’s new urban developments (made of the endless repetition of highly simplified giant identical types) cannot evolve in time.

Resilience is not an urban quality that can be reached by a strategy at only one scale. It is rather a property that emerges from the relationships between scales. What matters more than the nature of urban elements is the structure of urban connections. In a multiply connected, living organic structure, the smaller components can be changed without affecting the overall structure. Building the whole from the parts in an organic way leaves room for evolution. Starting from the whole creates structures that cannot evolve. Modifying the urban whole once it has been established from a technical blueprint involves destroying a great many components on very different scales. In a bottom up city evolving through a myriad of micro processes, it is, to the contrary, easy to modify smaller components (Salingaros 2008). The more structured and complex the city, the more readily it can absorb the perturbations to which it is subjected, without letting them upset the stability of its structure. It is in assimilating the fluctuations and tensions that complexity grows.

Scaling hierarchy and market fluctuations

Scaling urban structures are more adaptive to economic instability and market fluctuations. They display a fine grain platting (subdivision of land into plots), with a great variety of sizes, some large plots and a long tail of small plots. The platting can be easily reconfigured, subdivided like in Tokyo or consolidated like in Manhattan. The variety of plot sizes provides investment opportunities for every budget and every investor, which creates a diversified market with a multiplicity of actors that increase the complexity of the urban system. The scale-free diversity of actors increases the number of potential interactions and supports businesses, innovation and creativity at all scales.

Scaling hierarchy and social diversity and integration

The American-type suburbia or the South African townships made of repetition of small elements (large villas in rich suburbia, matchboxes surrounded by shacks in townships) have not enough initial complexity to evolve into scaling structures. This lack of spatial and formal scaling locks the urban development into a fragmented metropolis with no social mix and prevents the emergence of socio-economic scaling. A feedback loop is then created where the absence of socio-economic scaling prevents the emergence of spatial scaling.

Scaling hierarchy and accessibility

A scale-free distribution of amenities ensures a general accessibility. In Paris intra-muros, scale free distributions enhance accessibility with a long tail of small elements. The distribution of parks, for example, is scale free with a small frequency of big parks (17 parks bigger than 7 ha), a medium frequency of medium size parks (65 parks between 1 and 7 ha), a high frequency of small public gardens (300 public gardens less than 1ha including 260 less than half ha). Healthcare, shops, leisure are also distributed within the urban fabric according to inverse power laws.

Manhattan original Commissioner’s plan comprised only 4 large parks while contemporary Manhattan has more than 300 parks. Manhattan today presents the same scaling hierarchy of its public parks as Paris. It ensures a general accessibility at less than 10 minutes’ walk. The scaling parameter is higher in Manhattan than in Paris, which reflects a steeper hierarchy and more inequality between big and large in Manhattan than in Paris. Manhattan plan originally comprised only a few large public parks. Under market forces, they were abandoned in favor of the creation of more numerous smaller parks in order to increase the land market value of the city.

Scaling hierarchy and connectivity

In complex subway systems like in London, the number of connecting lines in metro stations (that is the degree of the nodes in the graph of the subway system) and their flows of passengers follow inverse power laws. Complex street patterns have also scaling properties (Bourdic and Salat 2012).

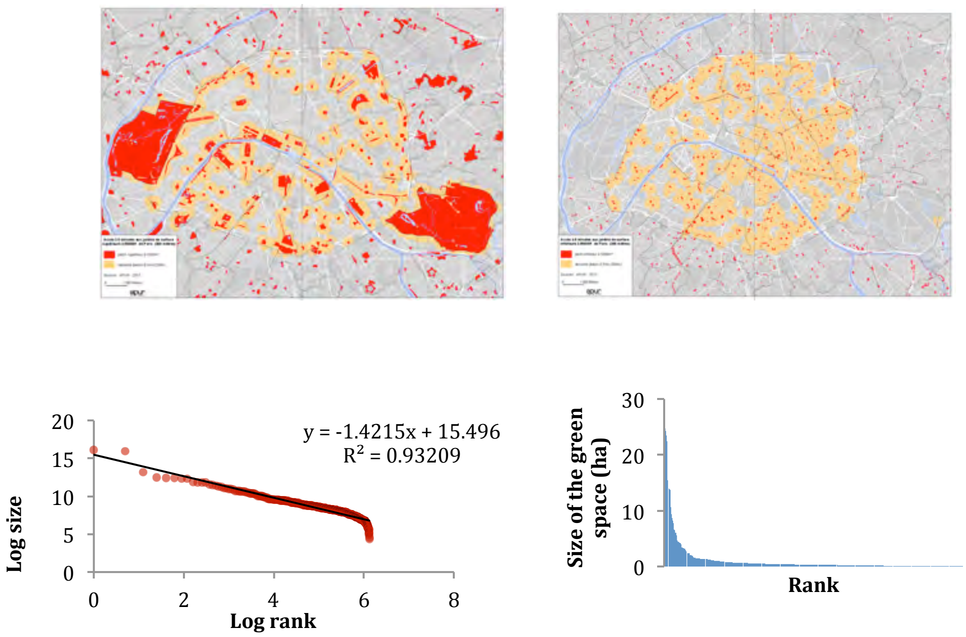

Figure 3: 300m accessibility to large public parks (more than 5000 m2) in Paris 300m accessibility to the long tail of small public parks (less than 5000 m2) (top), and Rank-size distribution of green spaces in Paris: The long tail of 260 public gardens less than half ha ensure general accessibility.
(Source: Urban Morphology and Complex Systems Institute).

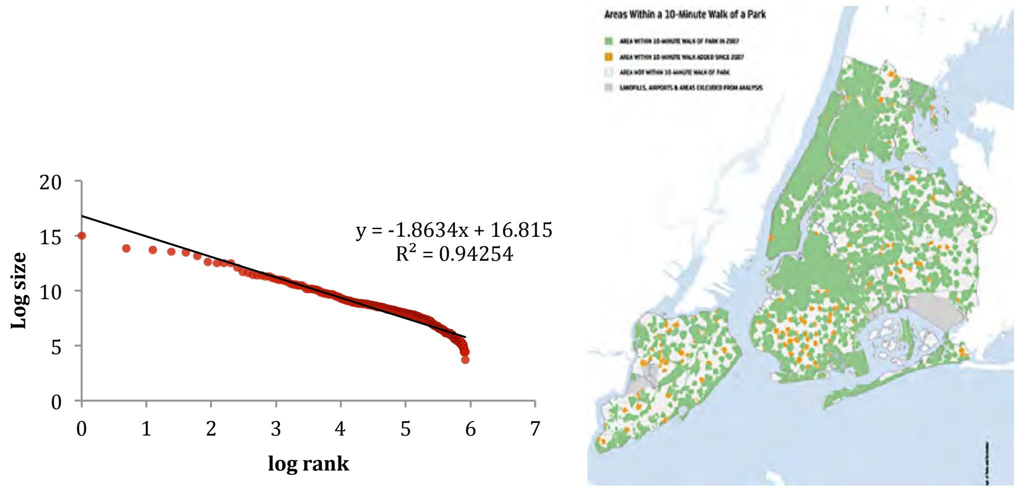

Figure 4: 10min walk accessibility to green spaces in New York City (bottom) and scaling properties of the distribution (top) (Source: Urban Morphology and Complex Systems Institute)

The scaling properties of different sub-systems in complex cities are coherent one with the other. The map above shows the frequency of buses in Paris along main streets over-layered on the map of public gardens. Streets are scaling, frequency of transit is scaling, gardens are scaling. Buses ensure accessibility to the larger amenities along main transit lines while the long tail of smaller streets ensures accessibility to smaller amenities. The different scales are well integrated.

URBAN EVOLUTION IN PARIS HAS CREATED A MULTI-FRACTAL FINE GRAIN PLATTING STRUCTURE EMBEDDING THE MEMORY OF 2000 YEARS OF HISTORY

Paris is not a city planned from the beginning. It is complex, connected, and highly differentiated while being integrated. Its urban form results from a balance between political and social power struggles, and market forces. The land division into parcels has maintained the historical continuity of the most ancient parts of the city (and now the most modern, vibrant and bustling of economic activity).

Paris land was, from the early Middle Ages and until the French Revolution, a multicellular city with intense competition between the cells. This fragmentation of land and power came from the feudal system. The land was divided between many Lordships. The Lords gradually granted to individuals settled on their land tenures on which they perceived an annual fee, the “cens” - hence the name “censive” for Paris Lordships. This property tax recognized the eminent property of the Lord on the land, the tenant having to settle for the useful property of the plot. This eminent property gave a number of rights to Lords: land rights such as the perception of the “cens” or transfer duties, but sometimes political rights such as rights on roads or high, middle and low justice. The Lordship fact was therefore an essential element of Ancient Regime urban life, a framework within which social life took place. The Lords started to divide their domains into plots and develop the land from rural to urban as early as the beginning of the 13th century, a process that started in New York only at the beginning of 19th century. Contrary to Manhattan, where the land division occurred within a unified global Euclidean grid covering the whole island, in Paris this process occurred within the fragmentation of powers of the feudal structure. It was an intense competition both for wealth and power in a highly fragmented society where the king controlled only a very reduced part of the capital.

The medieval land subdivision is the result of the city multicellular growth from successive subdivisions of noble and ecclesiastical censives. The successive morphogenetic ruptures of the medieval fortified walls created asymmetries still strongly visible in the platting five to six centuries later in the Napoleonic cadastre Vasserot of 1810-1836. In the early thirteenth century the abbey towns inside Paris did not really fit into an accomplished urban landscape. The towns inside Paris were formed very slowly. They filled up the gaps in the urban area, with heterogeneous structures at very different paces. The masters of the land (the bishop, the Convent, any particular noble or bourgeois) divided their remaining land into a number of units of roughly equal dimensions, regularly arranged along one or several streets. The Lords could receive higher profit and increased their influence and power because the new developments were targeted against the old cores, and provided economic and social benefits to new communities of men coming from elsewhere. The new developments were significant advantages in the struggle between the Lords of the land: canons against bishop, against Cathedral abbey, abbeys between them. Paris plot size pattern is a legacy of this medieval period, not of Antiquity: while the Roman period city is clearly focused on the left bank, the main urban center has developed on the right bank from the Middle Ages.

The geometric characteristics of past and present plot layers allow analyzing the urban morphogenesis: surface, elongation index (ratio length/width), index of rectangularity (the surface of the plot considered in relation to the minimum bounding rectangular box and the convex envelope associated with it). Overall, pre-industrial plots are in a range between 12m² and 300m², with plots most often between 50 and 100m². Highlighting below 300m² plots on the Napoleonic period Vasserot plan (1810-1836) confirms the high plot density on the more urbanized right bank compared to the more rural left bank. Per hectare there were on average 11 plots on the right bank against 8 on the left bank.

The plot analysis reveals 2 major perpendicular morphogenetic axes. The major orientation is between 60 and 74° with respect to east. It alone represents 36% of the total of segments. It relies on two morphogenetic axes (that can generate and transmit forms): the alignment formed by rue Saint -Martin and Saint- Jacques, and the Seine. Archaeologists have identified this orientation as dominant in the Roman period. The morphogenetic axis of ancient Lutèce was based on a regular orthogonal grid aligned on rue Saint- Martin – rue Saint- Jacques, which is partly the cardo of the ancient foundation and builds on former islands formerly present in the course of the Seine. This orientation also dominates the network of streets that existed at the end of the fourteenth century. The Middle Ages have played a key role in the resilience of Roman period main orientation and its dissemination on the right bank. This Roman and Medieval axis still structures the most innovative economy in Paris.

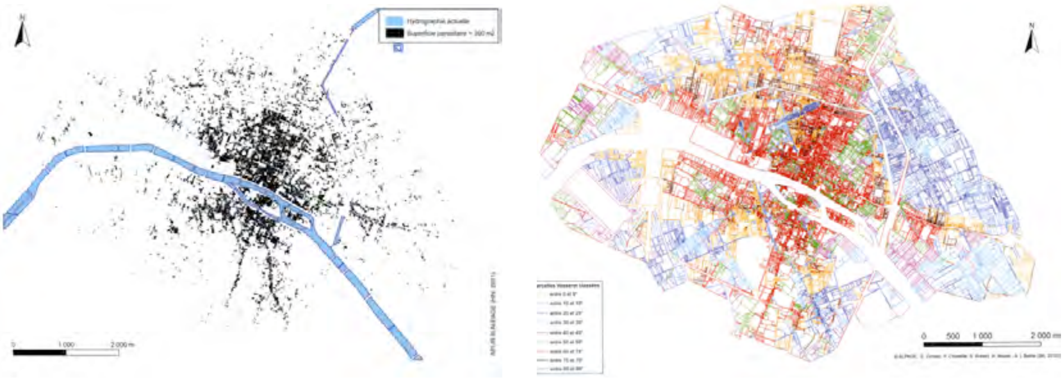

Figure 5: Extracting on the 1810-1836 cadastral plan only the smaller plots (below 300m²) reveals a fractal pattern oriented according to the 2 morphogenetic axes of Antiquity and the Middle Ages (left). The orientations of the segments of plots in Vasserot map (1810-1836) as well as of the archaeological structures of Paris (1810-1836) (right) (Source: Noizet, Bove, and Costa 2013).

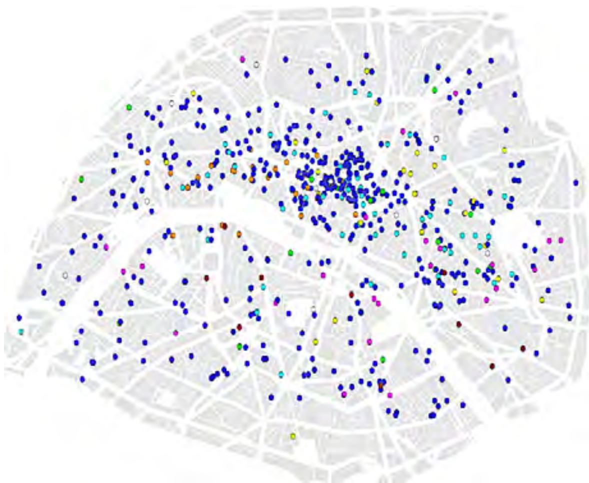

Figure 6: Map of the digital economy in Paris (localization of the ecosystem of start ups).
(Source: Urban Morphology and Complex Systems Institute).

Minute breaks of symmetry in the urban fabric have been transmitted through centuries by long-term correlations. Platting geometry (size, orientation) is a time travel machine in layered urban strata. It embeds the memory of the city at extreme micro scales. As an example, we can analyze the consequences on the platting geometry of the opening dates of gates in Philippe Auguste wall (1190-1215). When Philippe Auguste decided to build a wall, the king made clear his desire to see the whole enclosed area occupied by houses of new residents. Abbeys then conceded large censives to bourgeois who undertook the subdivision and installation of men. The Knights Templars decided to develop their censives in Le Marais still sparsely populated. Rue du Temple crossed the wall through Porte du Temple, one of the original gates. Rue Vieille-du-Temple was opened very early, before 1203. Rue du Chaume was opened only in 1288. The analysis of plots in Vasserot plan (1810-1836) reveals a morphological hierarchy with 20.3 plots per ha for rue du Temple, 15.5 plots per ha for rue Vieille-du-Temple, 11.3 plots per ha for rue du Chaume. The piercing of wall gates has been so structuring on the micro scale of the urban structure that 6 centuries after, at the beginning of 19th century, the spatial hierarchy of 13th century is still visible.

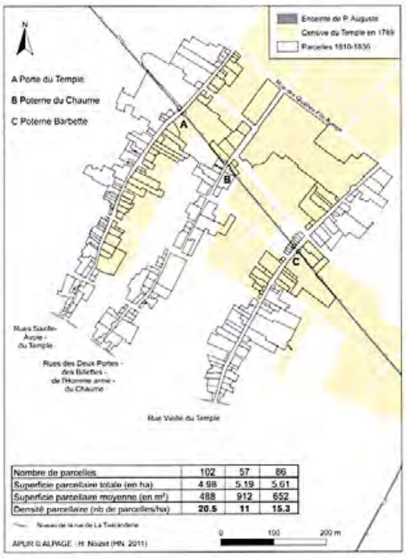

Figure 7: Platting at the beginning of 19th century (1810-1836) along 3 roads opened in the 13 th centuries. Local scale breaks of symmetry have been transmitted throughout 6 centuries. (Source: Noizet, Bove, and Costa 2013).

From this long term history with long-range temporal correlations emerged a multi-fractal urban structure, with local singularities and breaks of symmetry reflecting the stratification and imbrication of different morphological periods. Scaling hierarchy of plot sizes is the "signature" of complexity.

Figure 8: Scaling hierarchy of plots' area along a structuring axis of Medieval Paris: rue Mouffetard. The largest plot is 760 m2. (Source: Urban Morphology and Complex Systems Institute).

The evolution of Paris results from the superimposition of continuous, local growth processes and punctual changes operating at large spatial scales. The most important quantitative signatures of Haussmann planning are the spatial reorganization of centrality and the modification of the block shape distribution

(Barthelemy and Flammini 2008). In Paris intra-muros, Haussmann new boulevards (the “cuts”) reinforced the scaling structure of Paris street patterns by integrating the existing city into a larger scale-free structure.

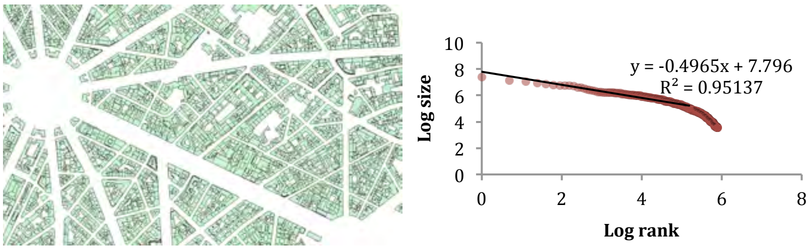

Figure 9: Quartier de l’Etoile. Plot scaling hierarchy in Haussmannian neighborhood developed 7 centuries after rue Mouffetard. The largest plot is 1600m². With much larger plots the scaling parameter remains almost identical. The city dilatation conserves the scaling hierarchy.
(Source: Urban Morphology and Complex Systems Institute).

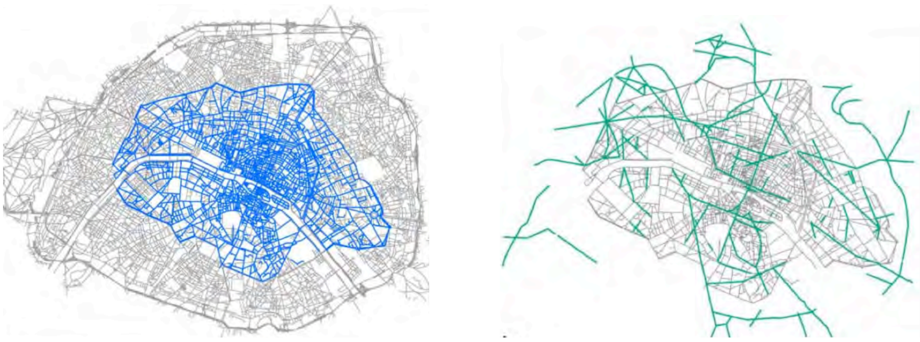

Figure 10: Map of Paris in 1789 superimposed on the map of current 2010 Paris (Source: Authors).

Map of Haussmann’s modifications: The grey lines represent the road network in 1836 (Plan Vasserot), the green lines represent Haussmann’s modifications. Recent studies (Barthelemy et al. 2013) have demonstrated that Haussmann’s interventions did not change the structural properties of Paris street network, except the reorganization of betweenness centrality. The usual network measures display a smooth behavior. The network has scaled up while keeping its fundamental characteristics. First basic measures include the evolution of the number of nodes N, edges E, and total length Ltot of the networks (restricted to the area corresponding to 1789). These indicators display a clear acceleration during the Haussmann period (1836–1888). The number of nodes increased from about 3000 in 1836 to about 6000 in 1888 and the total length increased from about 400 km to almost 700 km, all this in about 50 years. This node increase corresponds essentially to an important increase in the population. The number of nodes N is proportional to the population P and that the corresponding increase rate is of order dN/dP<0.0021(Barthelemy et al. 2013). Barthelemy and al. have plotted various indicators such as the number of edges and the total length versus the number of nodes taken as a time clock. The results display a smooth behavior. In particular, E is a linear function of N, demonstrating that the average degree is essentially constant and equal to 3 since 1789. The total length versus N also displays a smooth behavior consistent with a perturbed lattice.

These results on the smooth behavior of 19th century evolution confirm our findings on the scale-free continuous evolution of Paris street pattern. The frequency of streets of different widths in Paris (created during different morphological periods, Haussmann period being the latest large scale intervention) follows an inverse power law. In the long tail of narrow streets we find the streets 8 meters wide opened in the 13th century like Rue du Chaume between rue du

Temple and Rue Vieille du Temple.

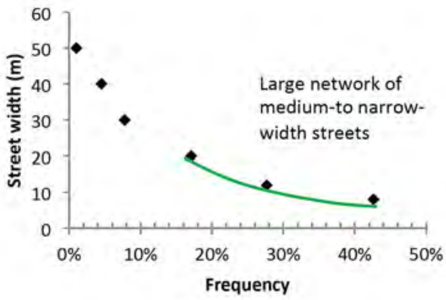

Figure 11: Scale-free distribution of street widths in Paris (Source: Bourdic and Salat 2012).

THE EMERGENCE OF A SCALE FREE PLATTING IN MANHATTAN UNDER MARKET FORCES

When discovered by Hudson in 1609, Mannahatta (« The Island with many hills ») had more ecological communities per acre than Yellowstone, more native plant species than Yosemite, and more birds than the Great Smoky Mountains National Park. Extreme ecological diversity has been replaced today by extreme human diversity. Before the grid, New York City grew organically. Concentrated at the Southern tip of the island was a knot of short streets, some dating back to the Dutch settlement of New Amsterdam, shaped by local conditions and lacking a unifying order. Most of the island was a patchwork of farms and meadows, ponds and marshes, laced with meandering country roads and providing ample ground for expansion. Towards the end of the American Revolution in 1776, the fundamentals of Manhattan, were almost unchanged since 2 centuries ago, except a town of 32, 000 inhabitants at the bottom of the island. After the American Revolution, the new and cash-strapped American city government looked to profit from its underperforming domain (about 2 square miles of rocky, hilly undesirable land in the middle of the island. The Common lands were vacant land first granted by Dutch provincial authority to the government of New Amsterdam in 1658. To facilitate their sale, Goerk prepared a subdivision plan with 3 long parallel streets, which would eventually become 4th, 5th and 6th avenues, with an east-west length of blocks identical to the one in Goerck's plan. This plan started the rise of New York real estate market and ascent of land values.

In 1811, the Commissioners' map overlaid a seemingly uniform grid of rectangles over the rugged island. The grid was above all an easy format for the subdivision and development of land. The grid system stripped the land of topographical markers and specificity, and repackaged it as standardized building lots. The grid re-conceptualized the island in a real estate market. And it worked beyond all expectations. In 1807, the assessed value of New York City real estate was $ 25 million. In 1887 it was $2 billion, a 80-fold increase.

From a seemingly homogeneous grid of blocks overlaid on a rocky, hilly, inhospitable island partly covered with marshlands, how did highly differentiated neighborhoods emerge, with urban and social fabrics as different as Soho, Tribeca or the Upper East Side? In reality the grid contains 2 patterns that create variety. One pattern is formed by the street widths (30 meters for the avenues, 20 meters for standard cross streets, with 15 major cross streets 30 meters wide at irregular intervals. The second pattern derives from block dimensions. All blocks are 60 meters wide north to south, but their length east to west varies diminishing from the center to the shorelines. From Third to Sixth Avenue blocks are 280 meters long. Moving eastward they shrink 189, 198, 195 meters long. Moving westward, they shrink uniformly to 244 meters long. Manhattan avenues are at a higher level in the hierarchy than Manhattan streets: first by their metric properties; second by their topological properties. Avenues are about 12,5 km long and 30 meters wide while streets are around 2.7 km long and 20 meters wide, except 15 major cross streets that are 30 meters wide. This break of symmetry in the pattern creates a metric scaling.

But even more important is the break of symmetry in the topological properties of avenues and streets. Graph theory defines street continuity by the number of links between nodes (segments of streets between intersections). It defines a street connectivity by the number of other streets it connects (that-is it intersects). As Manhattan avenues connect 155 streets and are made of about 144 blocks, while Manhattan streets connect about 11 avenues and are made of 10 blocks, there is a steep topological scaling between avenues and streets.

The scaling hierarchy in Manhattan street pattern comprised originally only 2 main scales (or 3 if we create an intermediary category for the main cross streets). This has been enough for the network to evolve in 200 years. The initial hierarchy with 2 levels has been transformed into a hierarchy with 4 levels, with towards the lower level small streets cutting through some blocks, and towards the upper level urban highways circling the island. This increase in hierarchy is reflected in the topological modifications of the grid. On a surface of 35.4 km2, the number of nodes of degree 4 characteristic of the pure grid has remained stable (1592 now compared to 1460 in the Commissioner's plan) while the number of odd nodes (degrees 1, 3, 5 characteristic of singularities and of more complexity) has doubled from 369 to 670. The density of nodes has increased (+ 17% to reach 60 intersections/km2) and the density of links has also increased (+21% to reach 11/km2) but they remain much lower than in a complex network like Paris. Sustainable networks must achieve a right balance between complexity (which reduces connectivity) and connectivity (which reduces complexity) (Marshall 2005; Salat 2011). The highly connective Manhattan grid has above all grown in connectivity compared to the Commissioners' plan by addition of new avenues (Lexington and Madison, which have divided in 2 the longest 280 meters blocks) and of Broadway diagonal.

Blocks were subdivided for land sale into identical plots of 205m2 area, which, under the influence of market forces, started to consolidate and create a differentiated platting ordered by combinations of the same basic module very early in the process. An example is the strategy of Charles Moore for developing his estate, which eventually became the vibrant and differentiated Chelsea neighborhood. Free market is a formidable time accelerator for differentiation and emergence of scale free structures. Dating as early as 1835, the map above illustrates the strategies of sellers of vast estates. Charles Moore developed his estate into Chelsea village, centered on Chelsea Square he had donated to the Episcopal Church in 1819. The break of symmetry created by the Square, Church and public garden created a cascade of differentiation in the size and value of the plots in relation to their location near or far from Chelsea Church. In 1820 Moore had evaluated his estate at $ 17,000. His wealth was estimated at $ 350,000 in 1845 and $ 600,000 in 1855, that is a multiplication by 35 in 35 years. Differentiation and asymmetry in land prices occurred very quickly in the seemingly uniform Manhattan grid. In 1860, real estate along Fourth Avenue ranged from $ 3,500 to $ 8,000, while lots along Madison Avenue were valued between $ 18,000 and 55,000 at proximity of Madison Square.

The breaks of symmetry in plot size and land value have created an enormous potential of differentiation. The most impressive diversification, which has ensured the vitality and enduring success of Manhattan, has occurred at the plot scale. In a scale-free morphological field like the grid and platting system of Manhattan, the position and the form of each element are influenced by its interaction on different scales with all other elements. When the result of all these interactions creates a form, it is neither symmetrical nor fixed. It displays a degree of plasticity that allows it to evolve.

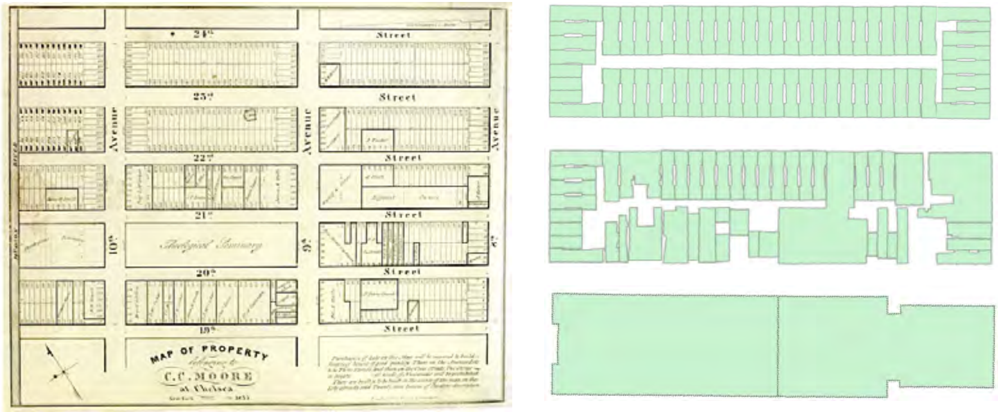

Figure 12: Map of Charles Moore real Estate in 1835, which eventually became Chelsea (top), and progressive consolidation of plots in Manhattan, from the original plot subdivision in 1811, with an average plot size of 205 m², to an intermediary plot consolidation, with an average plot size of 255 m², and an extreme plot consolidation, with an average plot size of 6,100 m² (Source: Authors).

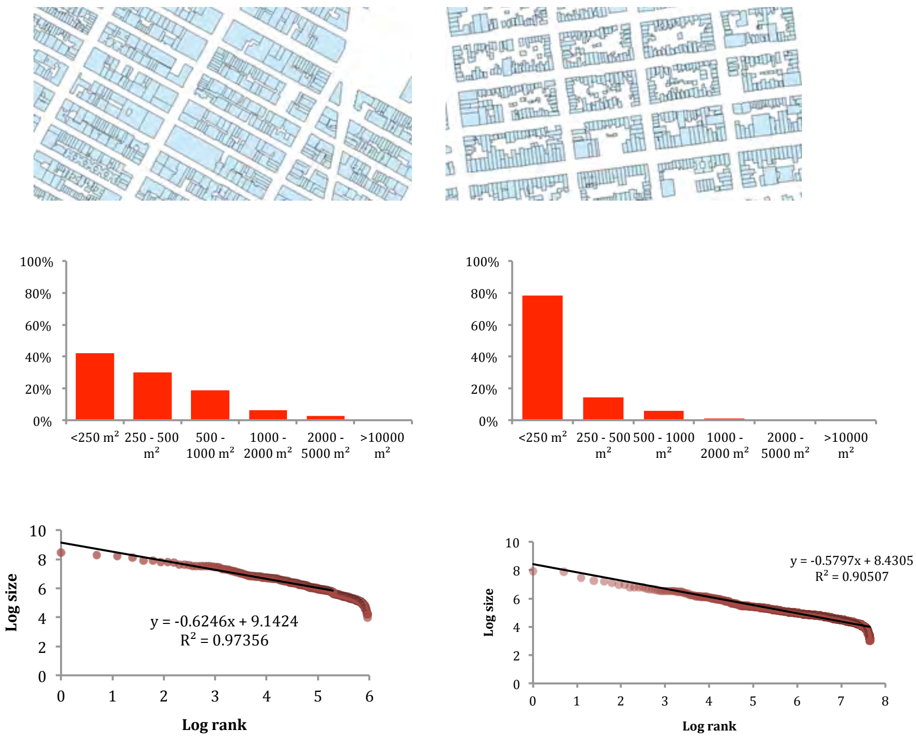

Figure 13: From an identical small-scale modular unit, the platting in Manhattan is highly adaptive: left around Madison Square, right in more residential Brooklyn, with corresponding rank size analyses
(Source: Urban Morphology and Complex Systems Institute).

Highly adaptive Manhattan platting follows an inverse power law with a scaling parameter higher than in Paris (0.6 compared to 0.5), showing a steeper hierarchy of scale. It is interesting to note that the scaling parameter in Lower Manhattan, which has the much longer history of being formerly the Dutch town of New Amsterdam, of which it has kept almost unchanged its street pattern, has a scaling parameter for platting of 0.5, characteristic of European cities like Paris.

40% of the plots around Madison Square have kept the original platting of early 19th century, while the other 60% have consolidated at various sizes. In more residential Brooklyn, 80% of the plot sizes date back to early 19th century. The plot sizes distribution inverse power laws allow for a large variety in the diversification of neighborhoods. In the very first decades after the Commissioners' plan of 1811, Manhattan vibrant emerging land market started to diversify the size and values of the plots, leading to a complex system.

This fast emergence of complexity of urban fabric, real estate market and economic activity in Manhattan lead to a multiplication by 8 of the population in 50 years making Manhattan as early as 1860 one of the largest city in the world with 800,000 inhabitants and to a multiplication by 80 of the real estate value of the city in 80 years (between 1807 and 1887). The emergence of complexity was fostered by breaks of symmetry in the apparently homogeneous grid and by a fine grain market of about 300,000 land plots of 205 m2 (quite interestingly the size of plots of South West of France medieval 12th century new towns called "bastides"). As a comparison on the same area of 66km2, Chinese recent urban developments display only about 250 giant superblocks; 1,200 times less plots than in Manhattan, which is even amplified by infrastructure oversizing compared to the Manhattan streets and avenues. This lack of fine grain restricts the market to 3 to 4 giant developers ("oligopsony") and prevents the emergence of a free market for land with a diversity of actors. This lack of market mechanisms is responsible for the economic failure of these new Chinese developments and their transformation into ghost towns with 64.6 million empty homes while 260 million urban migrants are waiting for decent housing. In New York on the contrary, 2 centuries of complex uneven growth and intensification have led to an extremely bumpy multi-fractal urban landscape for demography, development, energy, and most urban parameters.

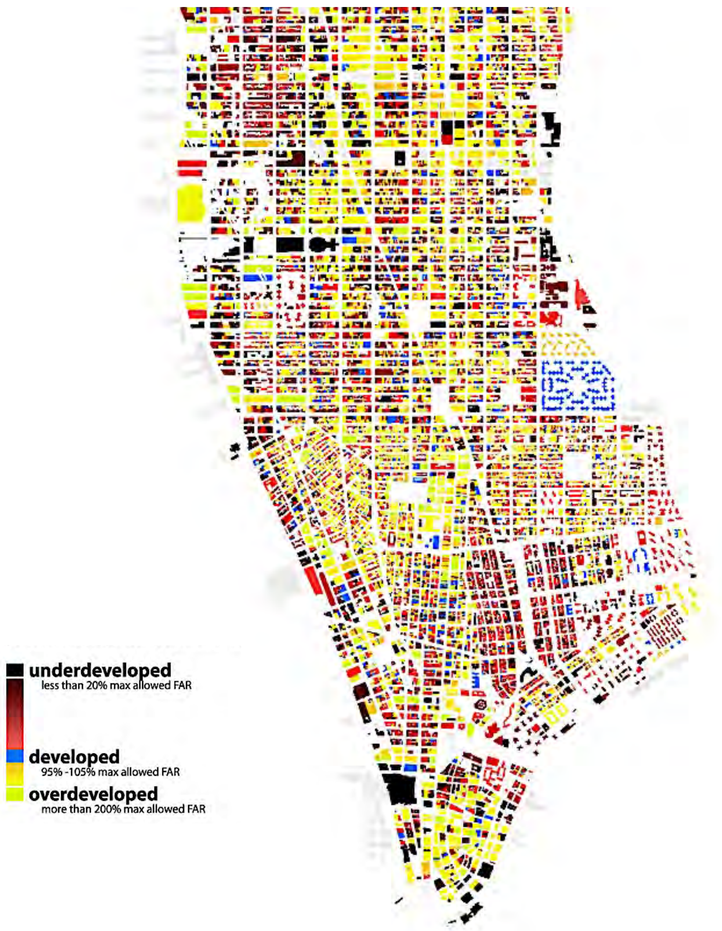

Figure 14: Manhattan uneven development (levels of FAR development of Manhattan blocks) (Source: Authors).

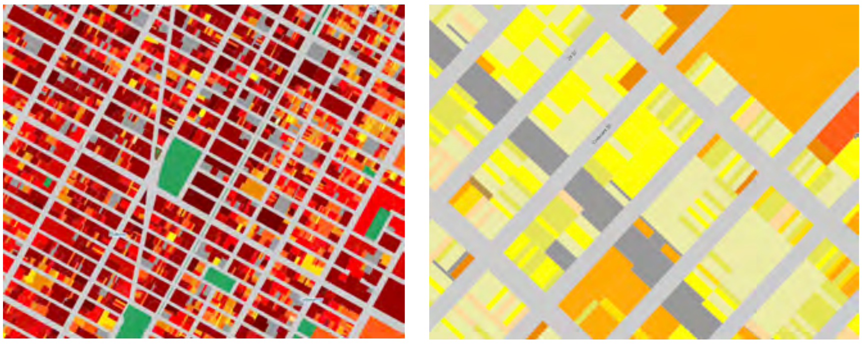

Figure 15: Manhattan detailed energy map reveals the fractal landscape of energy densities in in kWh/m² at plot level. The energy density at the tax lot level (in kWh/m²) varies more than 100-fold (Source: Howard et al. 2012)

150 YEARS OF COMPLEXITY GROWTH WITHIN THE EUCLIDEAN GRID OF CERDÀ'S DEVELOPMENT PLAN FOR BARCELONA

Unlike Manhattan, Cerdà's plan for the extension of Barcelona (Example) is a homogeneous and isotropic grid. No direction is privileged: all the blocks are square (113.3 meters side) and all the streets are 20 meters wide except the diagonals and a very few main long-range connecting roads. Cerdà plan led to dividing 13 km2 into 550 identical blocks. This lack of intermediary scales in the hierarchy has incited in recent years some planners to propose a functional reorganization with "superquadras" (superblocks comprising nine square blocks). Instead of adding a larger scale of superblocks, the natural bottom-up evolution of the city has been to develop, and intensify the smaller scales of the street pattern with a denser network of small narrow streets. From a "flat hierarchy" when planned, the scale-free properties of the Example have increased. On the 13.4 km2 of Cerdà's plan, the number of links per node has increased in 150 years by 45% (from 845 to 1,223). Nodes with 3 links have been multiplied by 2.4 (from 172 to 415). Nodes with 5 links have been multiplied by 2.3 (from 18 to 42).

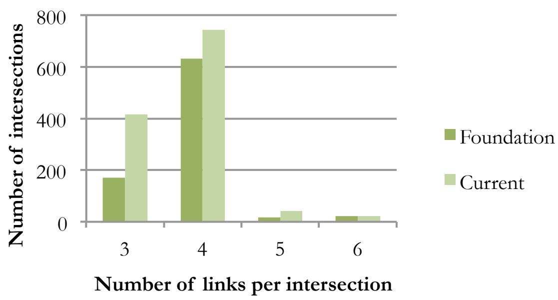

Figure 16: Number of links per node in Barcelona's Example from Cerdà's plan to now (Source: Urban Morphology and Complex Systems Institute).

This increase of the number of odd links per nodes reflects an increase of breaks of symmetry compared with the initial regular plan with a majority of even nodes (4 for the nodes on the grid, 6 for the nodes on the diagonals perfectly integrated in the grid), which represented 77.5 % of the nodes in Cerdà's plan and represent now only 62.6 %. The 20 meters wide "carrers" planned by Cerdà remain anyway dominant in the metric properties of the network: 180.6 km on a total of 222.1 km in Cerdà's plan to be compared to 161.2 km on a total of 228.1 km now. Smaller blocks have anyway resulted from the division of the large blocks, with the result to divide by 2 the standard deviation of the block size distribution compared to an inverse power law. We observe in 150 years a growth in complexity and connectedness of the network: the density of nodes increases by 44 % (from 63 to 91 street intersections/km2, still far from the more complex and connected network of Paris (circa 160), which results from 2000 years of evolution); the density of links increases by 40% (from 118 to 165). The cyclomatic number (that is the number of closed cycles in the network) increases also by 30% (from 55 to 76 cycles/km2, still far from cities with a longer history like Torino (117) or Firenze (156)). This increase in the cyclomatic number improves the connectedness and the pedestrian quality of the urban fabric. The average distance between intersections decreases from 126 meters to 105 meters, which also improves the walkability. The linear density of the street network is high (17 km/km2 to be compared to 22 in Paris and 3 in Beijing).

Measuring socio-economic breaks of symmetry with information theory

Information theory can provide a metrics of cities breaks of symmetry in diversity of uses and thus in complexity. Shannon and Weaver proposed an equation similar to the entropy equation to measure the quantity of information a message contains. They defined an information bit as the amount of uncertainty that exists when one has to choose between two possibilities: for each possible trajectory an information bit is added. If we consider that each legal entity of the urban system (economic activities, associations and institutions) is represented by an ideogram, a word in the urban dictionary, we can build urban messages and calculate the amount of information they contain. The value obtained (H) is the measure of diversity and of organized information. Diversity indexes have a greater meaning when applied to the temporal evolution of the urban territory. Studying the increases or decreases of H in a specific territory allows us to approach some of the potential dysfunctions of the urban system, as well as the elements that provide stability. When we evaluate the relationship between the consumption of resources (E) that are necessary to keep a specific organization (H) through time, we obtain an equation of efficiency (E/H) that may become a guiding function for urban policies.

A complex unequal socio-economic spatial structure overlaid on the physical isotropic field of cerdà's grid

Socio economic phenomena are not distributed according to Gaussian laws but to inverse power laws with high range of inequality. There is a contradiction between Cerdà's homogeneous plan and the structured scale-free diversity of economic activity destined to take place into it. For 150 years the spontaneous development of the city has increased the spatial scale-free properties of its physical form and of its network of connections, achieving an about 50 % increase of some key parameters from which the potential of interactions derives. This growth of complexity in the city structure and fabric has been provoked by the economic vitality of Barcelona and in return it has shaped the urban geography of socio-economic diversity. Despite the initial lack of spatial scaling, economic and social activity has made differentiation emerge along a major break of symmetry non predicted by Cerdà. Life has been stronger than top down planning and has chosen what Cerdà did not choose for the center of Barcelona. This break of symmetry has created a strong economic anisotropic morphogenetic field within the isotropic grid and it is also reflected in the scaling hierarchy of flows along transportation axes. Like in physics and like in semiotics, breaks of symmetry in cities create structure and meaning. Passeig de Gracia in Barcelona connects the original city that became the Barrio Gotico to the preexisting village of Gracia and is thus slightly shifted in the orthogonal grid of the city's extension without being a diagonal that would be integrated in the grid. It has been developed first in an astounding Art Nouveau Style. These two breaks of symmetry in space and in time have given Passeig de Gracia a central connective position and created a subtle large-scale order in the repetitive urban fabric of square blocks of Cerdà's Eixample. The crossing of the two diagonals that Cerdà intended to become the new center of the city did not break any symmetry and remained marginal compared to Passseig de Gracia.

Mapping urban Shannon information on a territory reveals the breaks of symmetry in the development of the urban fabric, and in some cases its elements of centrality. In Barcelona, we can see how the breaks of symmetry in information structure the apparently spatially uniform Cerdà plan. 34% of Barcelona's economic activity must be described by 6 bits of information, Eixample and the Barrio Gotico being the urban territory with a greater diversity. 87% of Example obtains values over 6 bits, becoming the fabric with the greatest diversity of legal entities (economic activities, associations and institutions) (Busquets & Corominas, 2010). Example has been built and is still being built slowly, as complex systems are built in nature. Example radiates activity around it; it is a true heart that beats diversity, extending urban complexity by the pedestrian axes that pass through it. The part of Example around Passeig de Gracia and the part of Barrio Gotico connected to it by Passeig de Gracia show the highest level of diversity and organizational structure in terms of information theory. They are the true heart of the city.

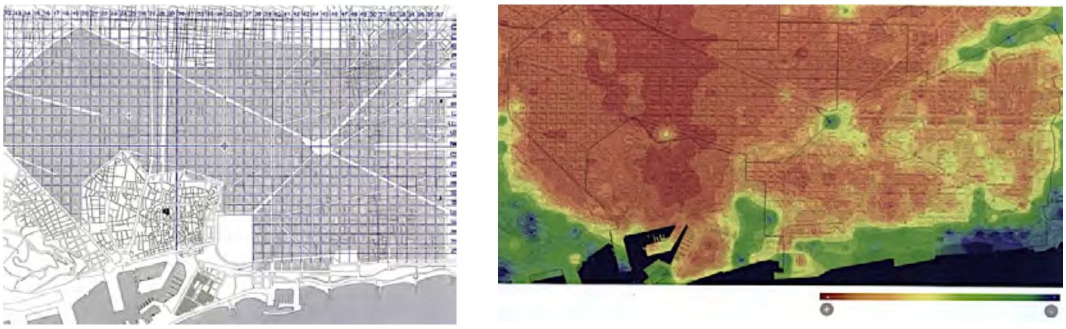

Figure 17: Left: Barcelona grid of square blocks in the Eixample with the Barrio Gotico. Right: Concentration of bits of information per individual (Source: Agencia d'Ecologia Urbana de Barcelona).

Connectivity increases structural information. On the contrary, when an axis presents a discontinuity in the linking of activities, the fabric is simplified, showing a hole in the diversity map. This is what happens at the crossroads of the diagonals that failed to become the centre of the city. The proposed designs for the Plaça de les Glories and its surroundings were put forward in terms of continuity and resolution of the traffic, but they became a barrier in the urban diversity continuous flow.

WHAT SHOULD PLANNERS DO AND MUST NOT DO?

Emergence is the opposite of the utopian simplified orders that architects such as Le Corbusier have tried to impose on cities. Huge quantities of energy are needed in such artificial repetitive orders to maintain urban systems in a stable state. Modernist cities, with abstract giant forms imposed from the outside, obstruct the emergence of small-scale connections, whereas the continuous creation of connections in historical cities favored their evolution. The continuous fabric of traditional buildings with courtyards, because of its connective forces developing inwards, has a stabilizing impact on the urban system. Giant modernist buildings standing in loneliness isolation do not connect into the urban fabric. They have a destabilizing impact and fail to create an evolving adaptive structure. Modernist architects turned their back to the universal laws of urban evolution by working with large-scale elements only and making the urban land a blank slate devoid of the incremental successive layers of historical traces. The Utopian machinist juxtaposition of vast homogeneous zones, made of a repetition of very big objects, hinders the appearance of emerging properties that were not integrated or even forecasted or predictable into the initial framework of the system.

Planners should create the framework for future evolution. They should not constrain this evolution. Successful plans are simple but subtle plans that leave ample room for unexpected and unpredictable change while enduring for millennia. Roman Empire plans lead to cities as different as Torino, Firenze, Bologna, and Paris. When the Commissioner's designed Manhattan map in 1811, long before the Industrial Revolution, in the period of Napoleonic wars in Europe, none of the technologies that made the power and wealth of the city in the 20th century could have been imagined: electricity, automobiles, subways and elevators where not only unpredictable, they were unthinkable, they were undreamt nightmares of a distant future. Without them, the typical urban form of Manhattan, the skyscraper, could not have been built. And yet, the Commissioner's plan was able to accommodate the unpredictable and to endure for 2 centuries, making Manhattan the world economic capital. Urban form does not follow function. It must successively or simultaneously adapt to many different and even contradictory functions. Planners should understand that the future cannot be controlled and that attempts to control it leads to dead cities, to ghost towns. Living cities are like chessboards where an endless number of different games can be played. It is the role and responsibility of the planner to design the chessboard, not to play the game. Life should play the game.

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