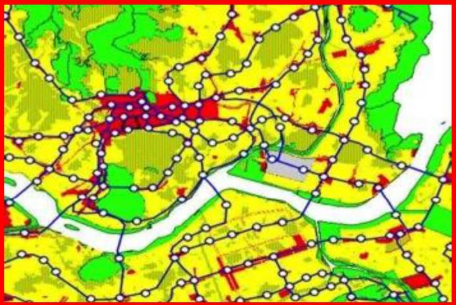
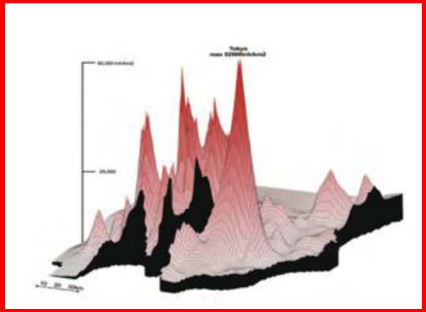
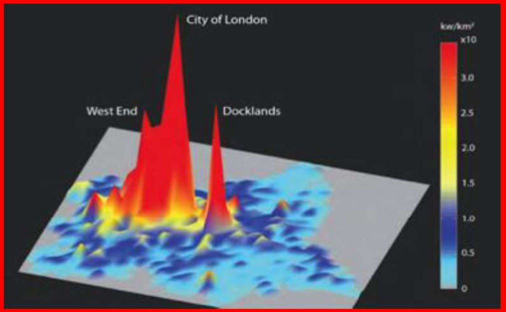
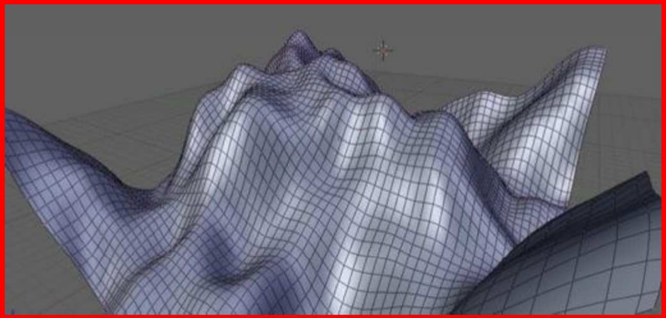





Urban Multifractals

Empirical facts and analysis framework
An urban historian and planner perspective



Serge Salat

Urban Morphology and Complex Systems Institute

May 7, 2014

Séminaire Cristolien d'Analyse Multifractale

2 billion people

will be urbanized by 2030

10,000 $billion/yr

will be spent in investment in infrastructures

Urbanization occurs at an unprecedented speed and scale, with a massive sprawl phenomenon

Urbanization is the key driver of economic growth (agglomeration economies) and of climate change acceleration.

With 700 million urbanites in China, 40% of the world middle class is now Chinese.

In 2030, China will have 1 Billion urbanites.

At the end of 21st century, there will be 9 Billion urbanites worldwide.

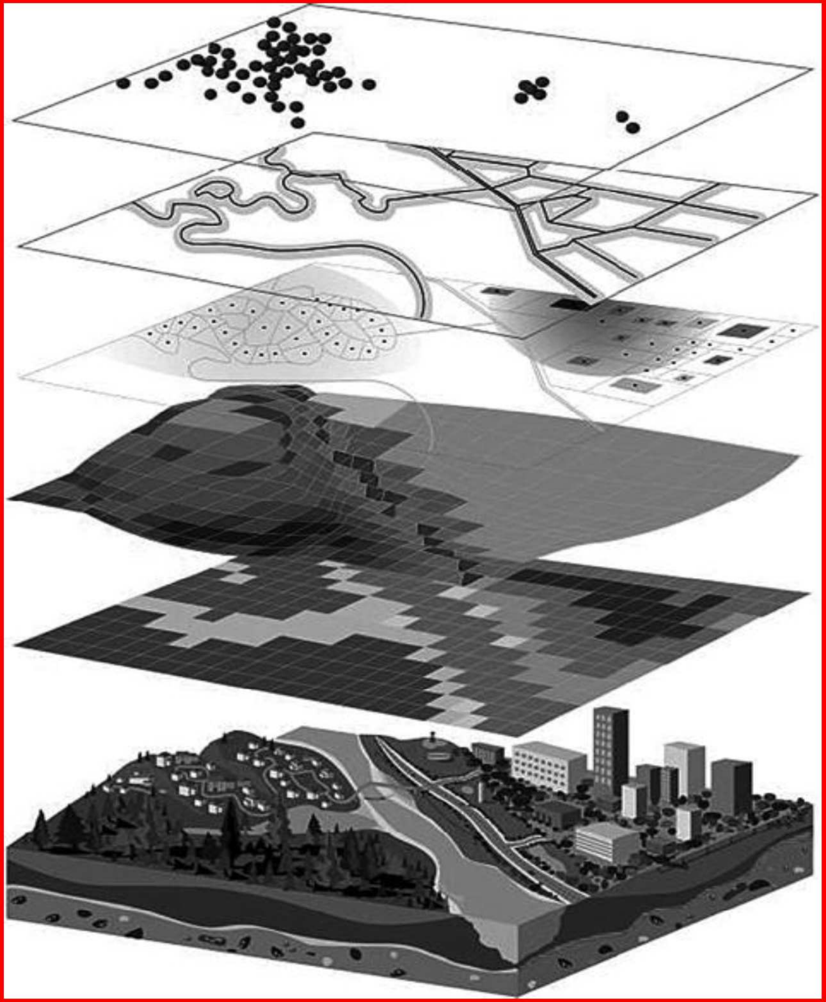
A science of cities is needed to better understand and pilot urbanization. We need more scientific analytical and predictive tools to elaborate scenarios for decision makers

(International agencies, governments, cities).

The challenge of a science of cities is to understand the links between urban form (multiscale and evolutionary) and economic, social, environmental (energy, etc.) efficiency and resilience (adaptiveness to progressive or catastrophic change).

The challenge is also to understand the relationships between self organization and planning. The large number and the diversity of agents operating simultaneously in a city suggest that cities are a multifractal emergent phenomenon ruled by self-organization. On the other hand, central planning plays an important role in the city, leaving long standing traces. Indeed central planning could be thought of as an external perturbation, as if it were foreign to the self-organized development of a city.

Urban form is the complex interplay of 6 layers (human and physical)



1. People (political, social, economic networks)
2. Streets and transportation networks
3. Platting (Plot division)
4. Topography
5. Land use
6. Buildings

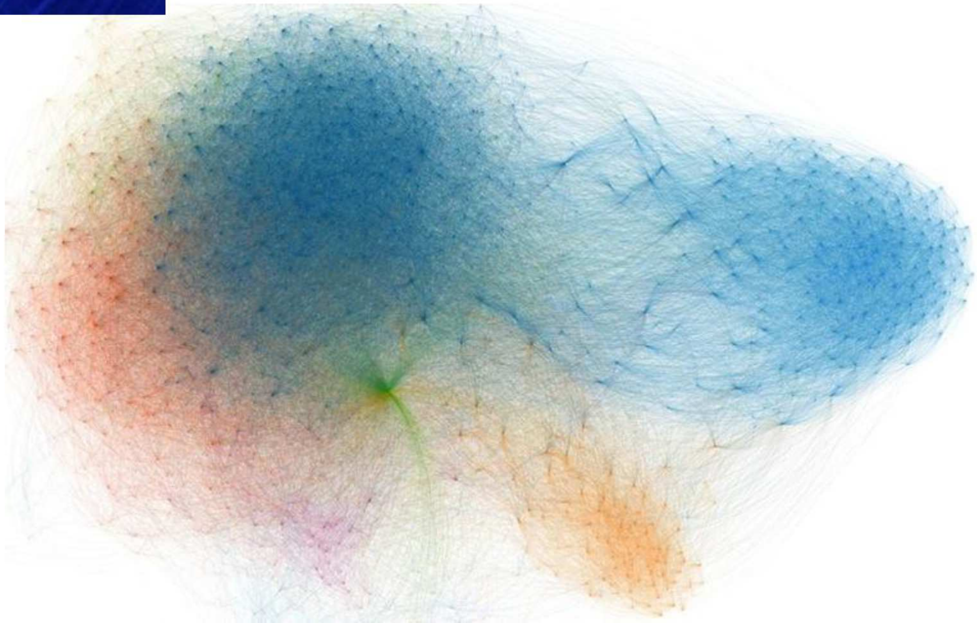
What are the relationships between layers of space, networks and human activity intensities
How do they evolve over space and time?

Each layer can be mathematically described with a set of morphological parameters

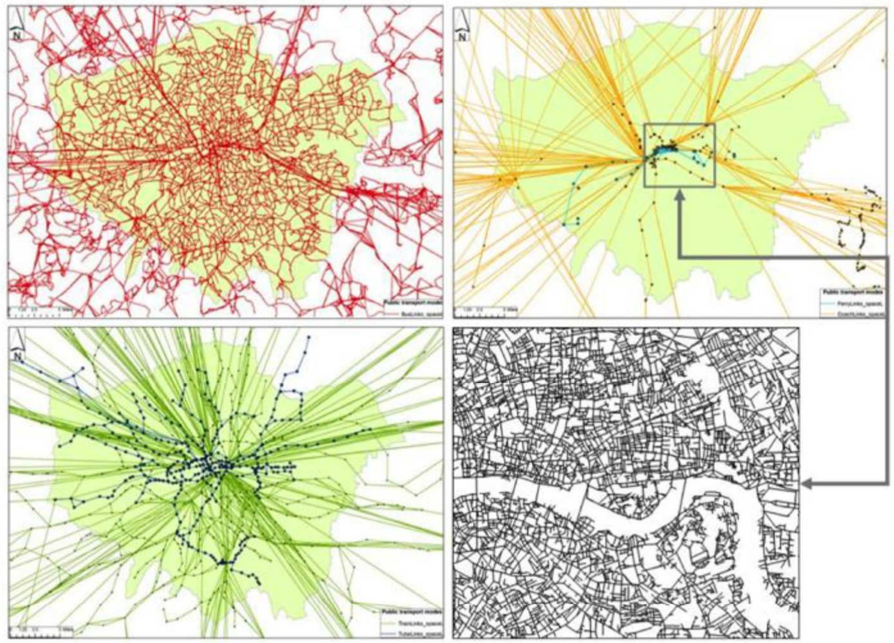
- Mathematical description of the first 2 layers: Graph theory
- Mathematical description of the 4 other layers: spatial analysis (Euclidean, rank size power laws, fractals, multifractals)
- Time series are necessary to study not only forms but also morphogenesis and evolution

Layer 1: Human activities are organized in networks that can be described by graph theory

The structure of Facebook



Layer 2: Street patterns are embedded in larger transportation networks



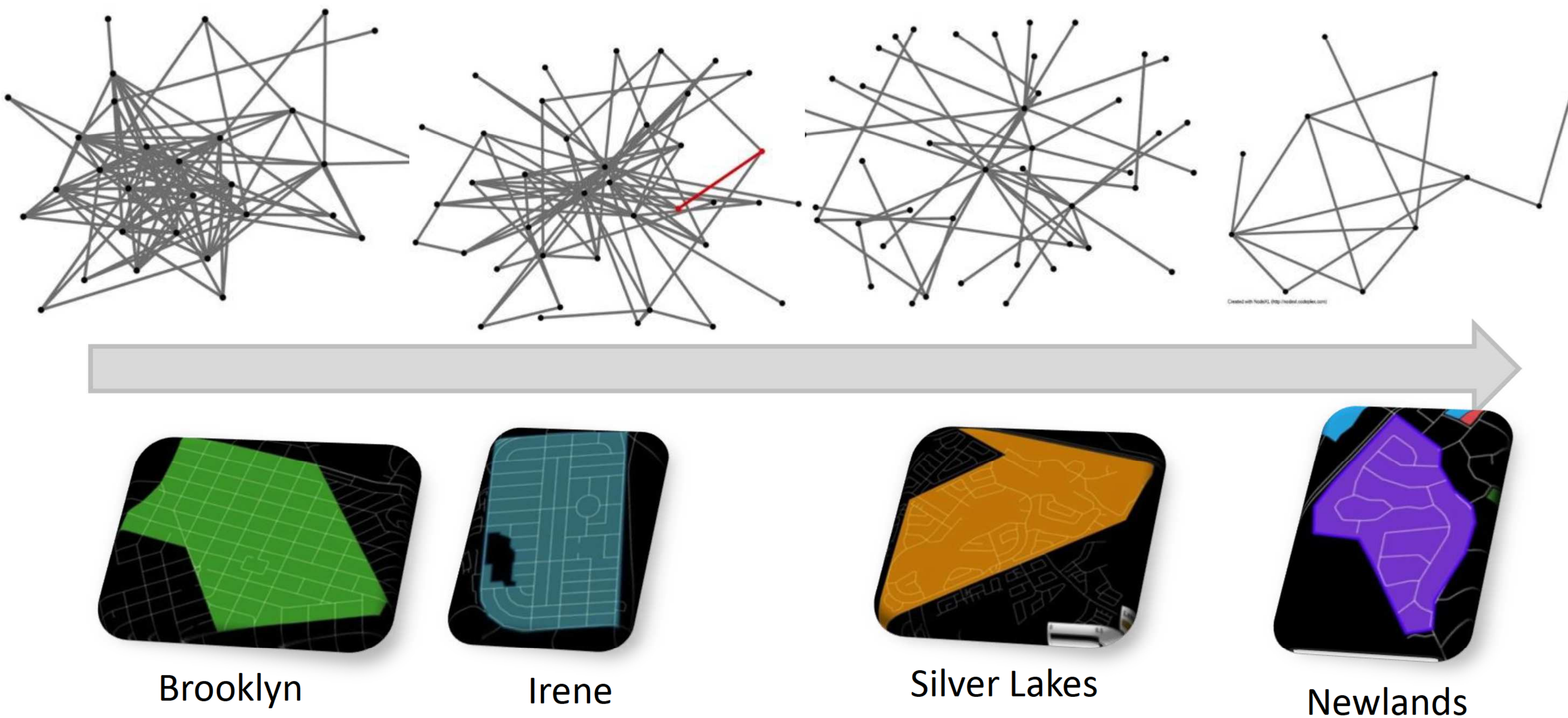
Coupled Transport Networks Generating a Convoluted Dynamics of Traffic

Top left shows bus routes (red) in London with the longer straight lines being the routes of long distance buses, top right shows intercity coaches (yellow) and ferries (blue), bottom left are long distance and overground rail (green) and tube (blue), and bottom right is a sample of the network in central London on which private car and taxi flows take place.

Source: Michael Batty, *The new Science of Cities*

Layer 2: Street patterns mirror social networks

The topological disconnection of street networks in South Africa reflects the replacement of a racial apartheid by a market apartheid (Tshwane/Pretoria)

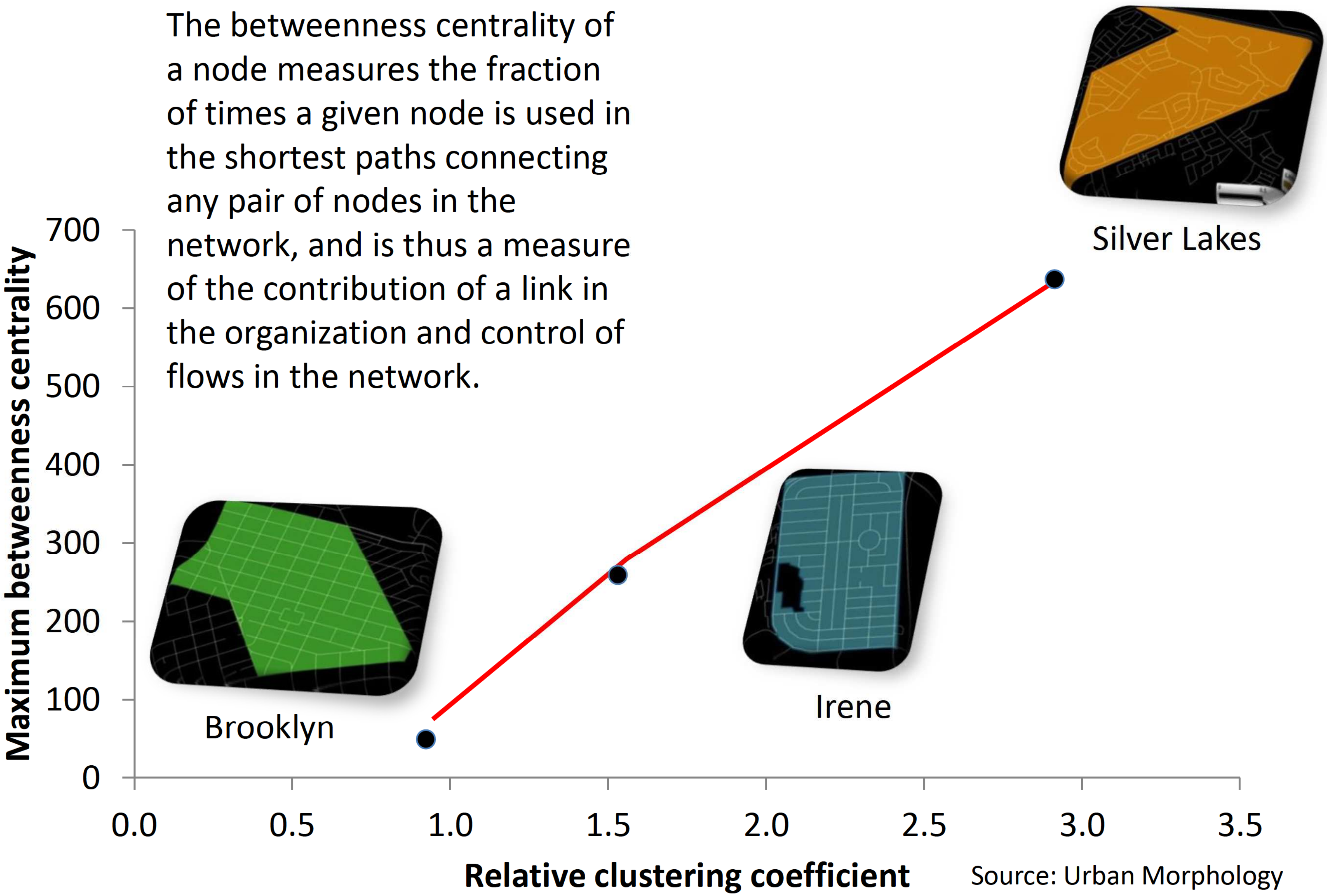


Above: Dual representations of street patterns where streets are nodes and intersections between streets are links. From open neighborhoods planned in 1904 (Brooklyn), to self enclosed post apartheid neighborhoods (Newlands).

Source: Urban Morphology Institute

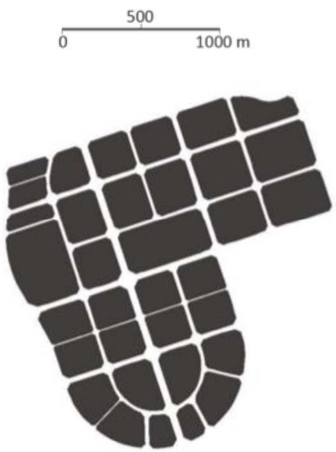
Layer 2: Street patterns mirror social networks

Increasing clusterisation and control in Tshwane/Pretoria reflect a fragmented society



Source: Urban Morphology Institute

Layer 3: Platting is the basic unit for land markets
Chinese cities have no multiscale fractal structure because of superblocks and the lack of fine grain platting



Chinese superblock
400x400m
Block size: 160,000 m²

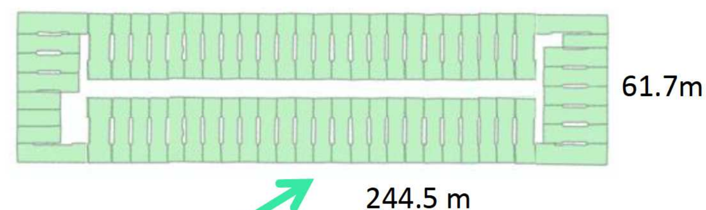
Block size should be divided by 11

Plot size in Manhattan Commissioners Plan is the same as in French 12th century South West *bastides*. Block size along Manhattan avenues is smaller than Roman empire blocks.

Subdivided superblocks
(Calthorpe design)
120x120m
Block size: 14,400 m²

Plot size divides by 70 the block size in Manhattan or Paris

Manhattan original plot subdivision
Block size: 14,800m²
Average plot size: 205 m²



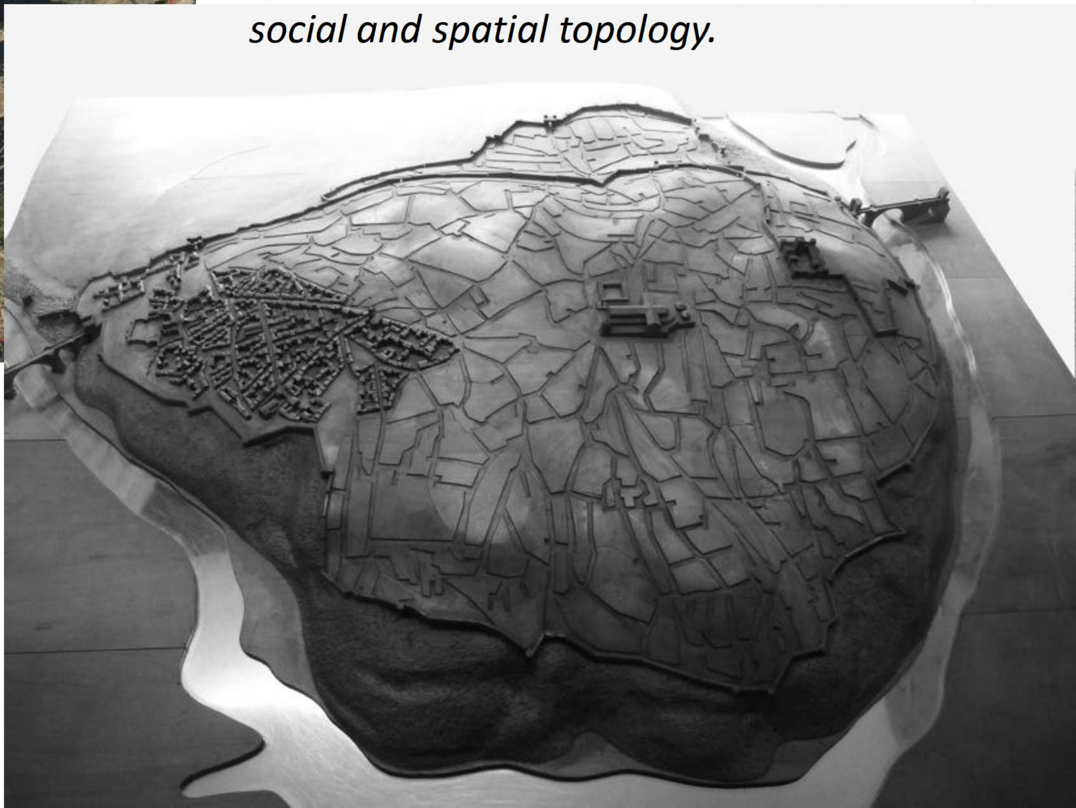
Source: Urban Morphology Institute

Layer 3: Platting is the long memory of the city (Kyoto, Tokyo, London 1666, Paris)

When the Great Fire of London burned out on September 5, 1666, it destroyed the city medieval core. Plans for rebuilding London reflected a common desire to replace London's narrow, twisting streets and fragmented medieval neighborhoods with a modern rectilinear vision. Christopher Wren imagined a new London with a regularized grid of rectangular blocks running parallel to the Thames, crossed by broad avenues radiating out from the rebuilt Saint Paul's cathedral. But although it had lost its buildings, London was not a blank slate. It was covered with property lines that the inhabitants wanted to maintain. London was rebuilt according to a pre-fire survey and kept its medieval structure, where now is located the most successful world financial district.

Layer 4: Topography is a key morphological field for urban structure : Toledo

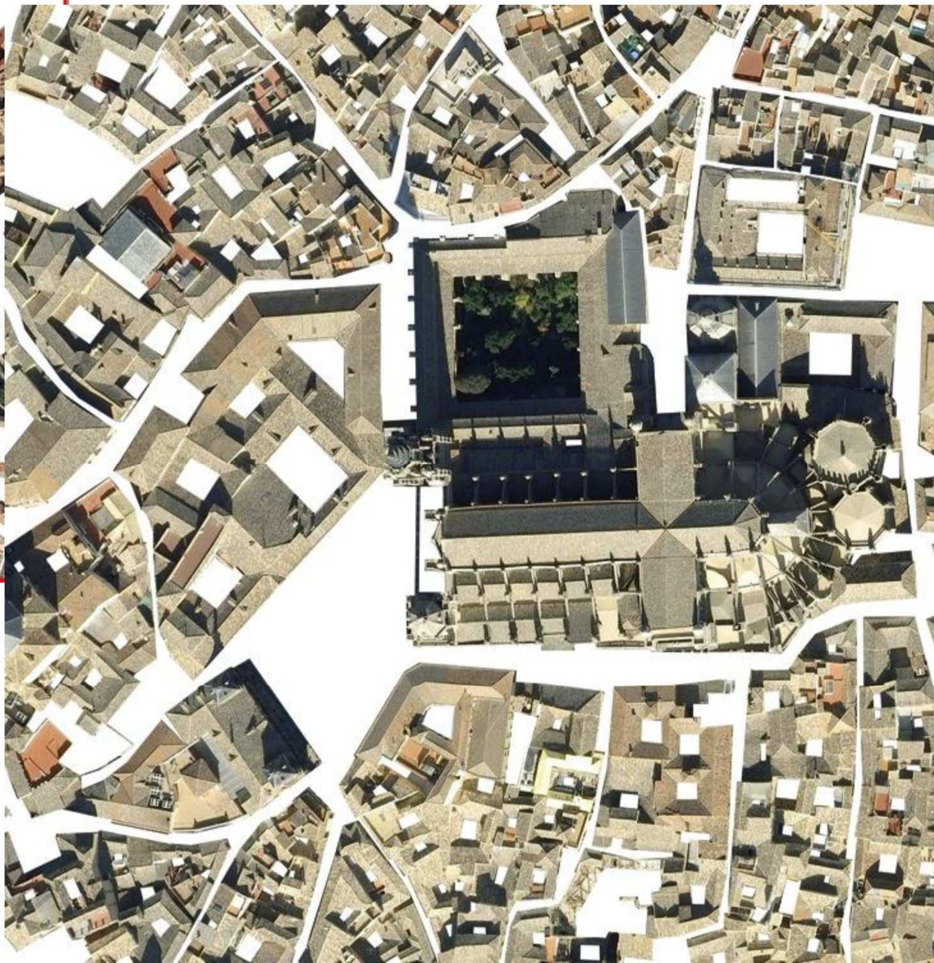
Three-dimensional plan of Toledo showing the complex irregular pattern of streets that resulted from the Christian reorganization of the Islamic street pattern. In 1492, the Chapter House of the Cathedral "flattened" to one level the ternary hierarchy of Islamic streets, made up of long souk streets leading to the doors of the mosque, secondary streets dividing up the big blocks, and a proliferation of dead ends providing access to interior spaces hidden in the heart of a segmentary social and spatial topology.



The divergence of lines is accentuated by the hyperbolic (saddle shape) geometry of the hilly topography that turns the city into a labyrinth. It is as if a folded, creased surface had made points that are actually very close in physical space more distant from one another.

Source: Serge Salat, *Cities and Forms*

Toledo layers: The Catholic footprint on an islamic urban fabric (1085-1492) previously Roman then Wisigoth
The Cathedral occupies the site of the great mosque

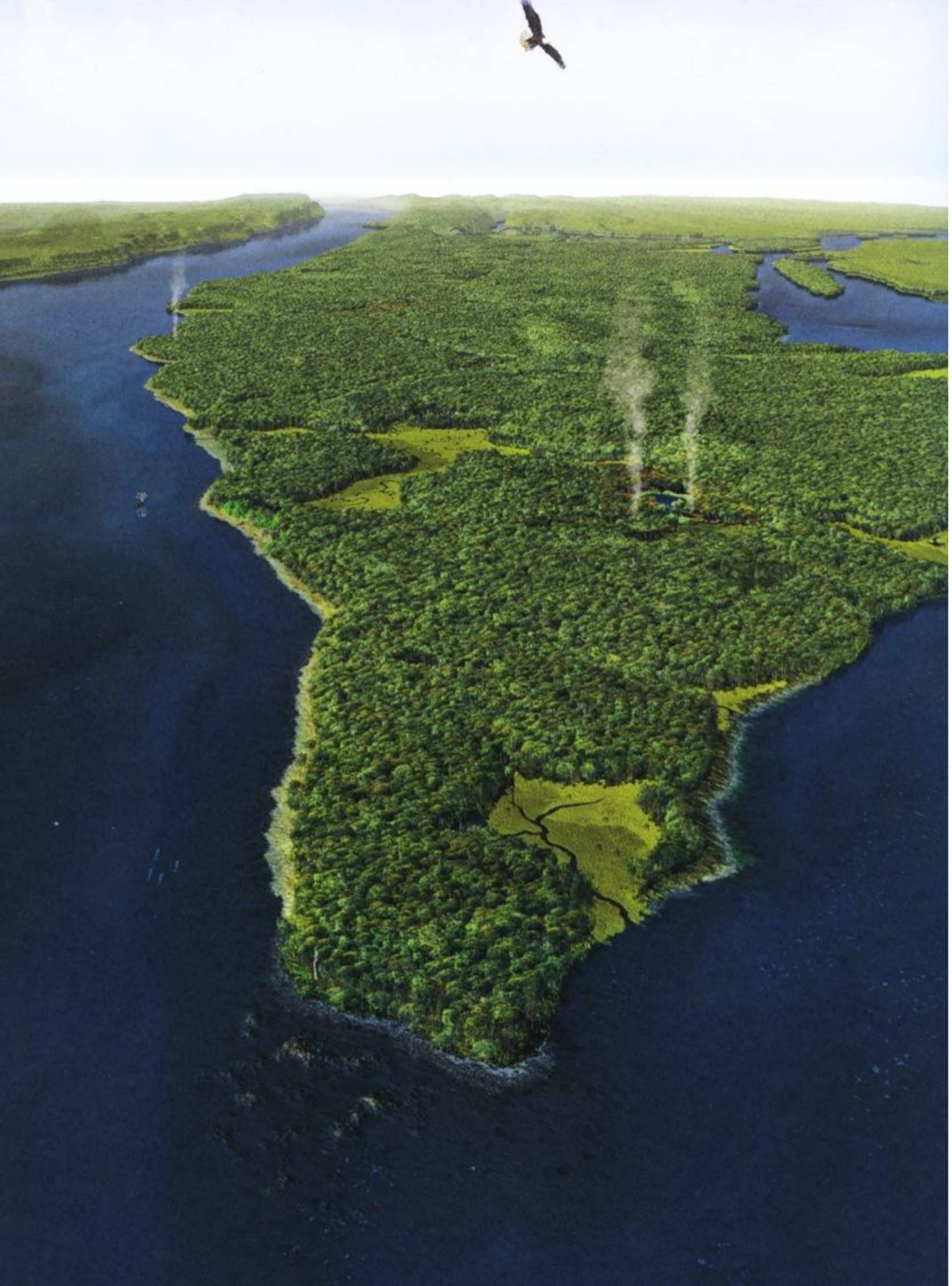


Source: Serge Salat, *Cities and Forms*

Layer 4: Venice, a multicellular city born out of water

Venezia Piazza San Marco 400 x 400

Source: Serge Salat, *Cities and Forms*



Layer 4: The original landscape of Mannahatta when discovered by Hudson in 1609

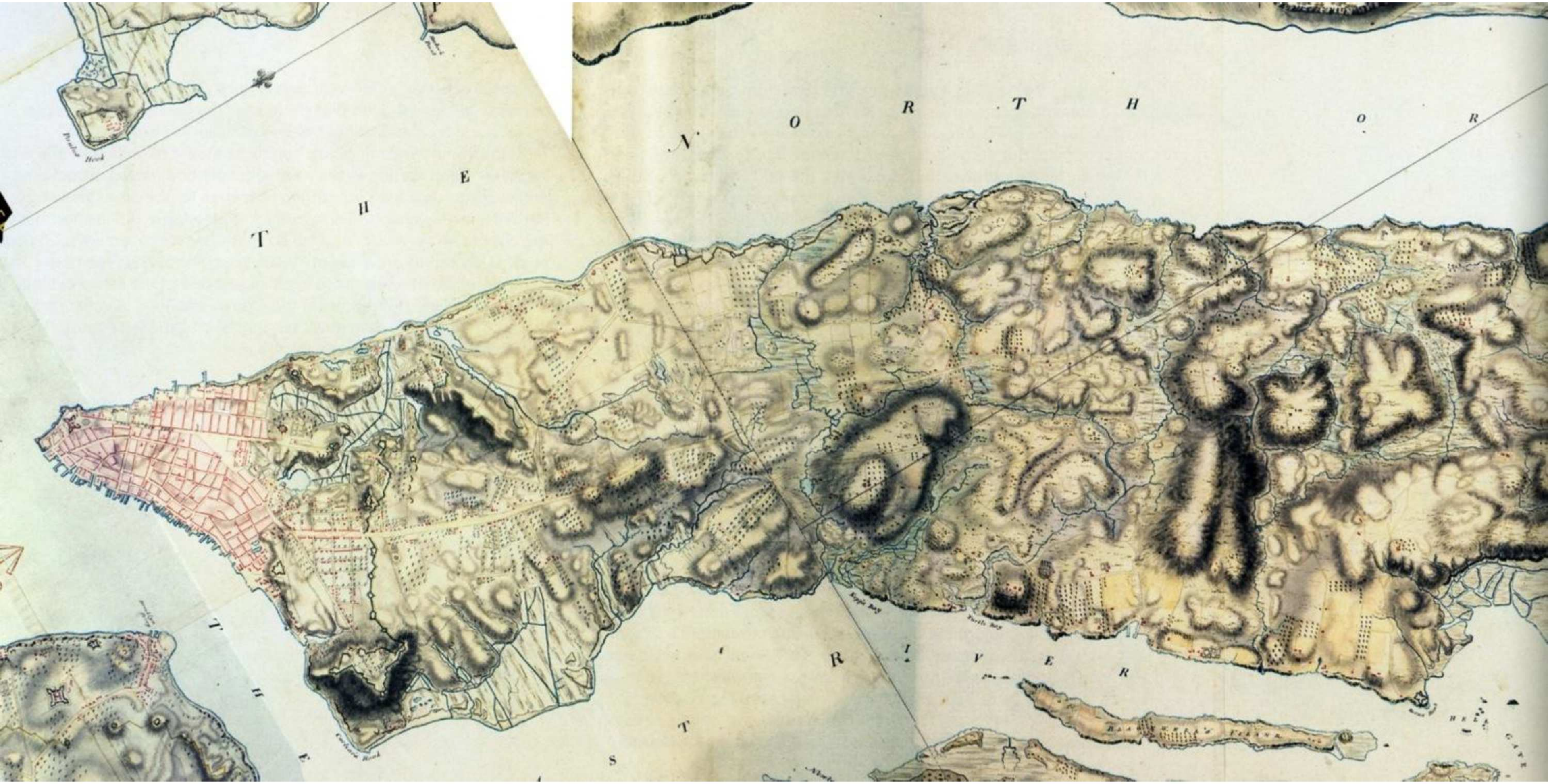
Mannahatta (« The Island with many hills ») had more ecological communities per acre than Yellowstone, more native plant species than Yosemite, and more birds than the Great Smoky Mountains National Park.

Extreme ecological diversity has been replaced by extreme human diversity.

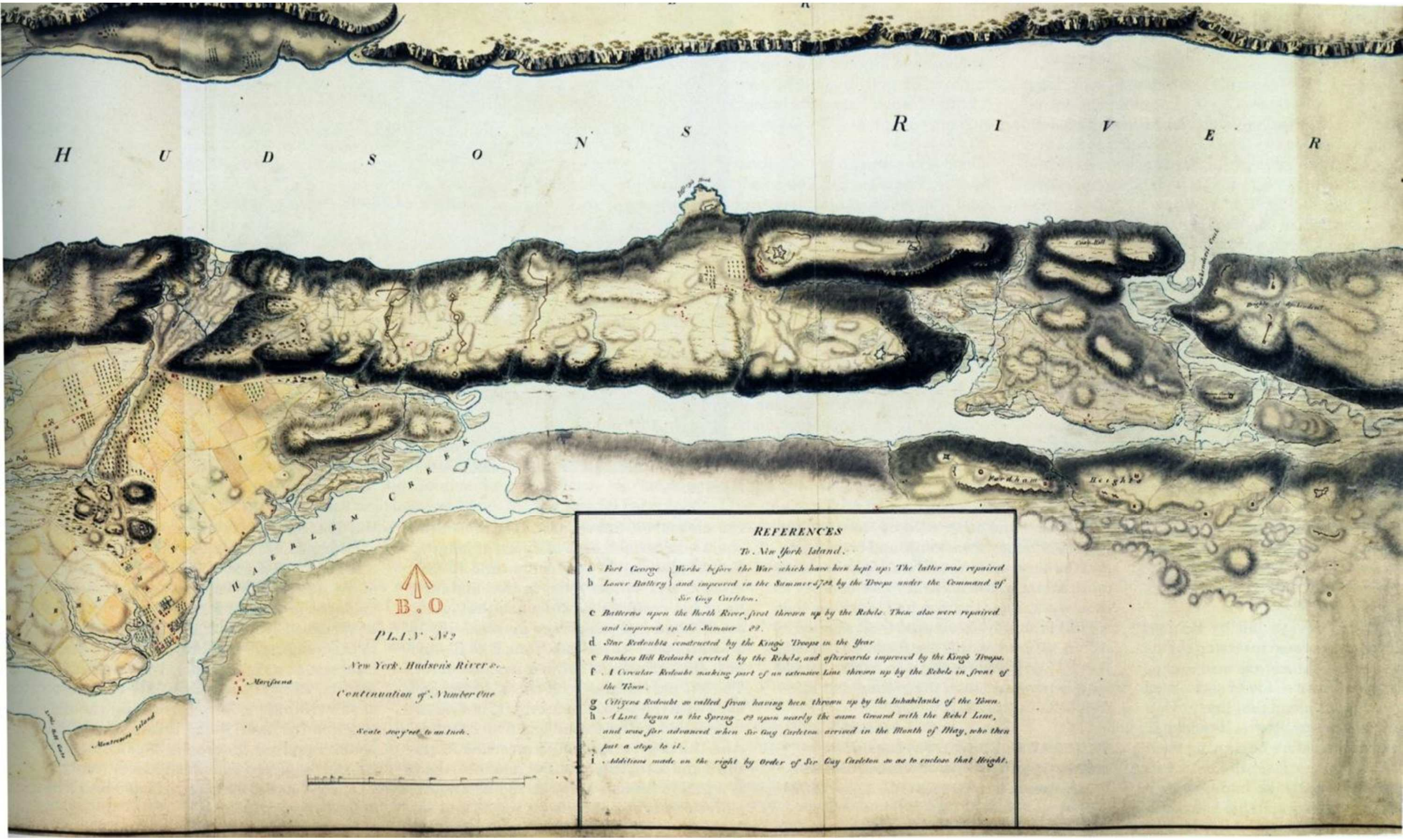
Source: Eric W. Sanderson, Mannahatta

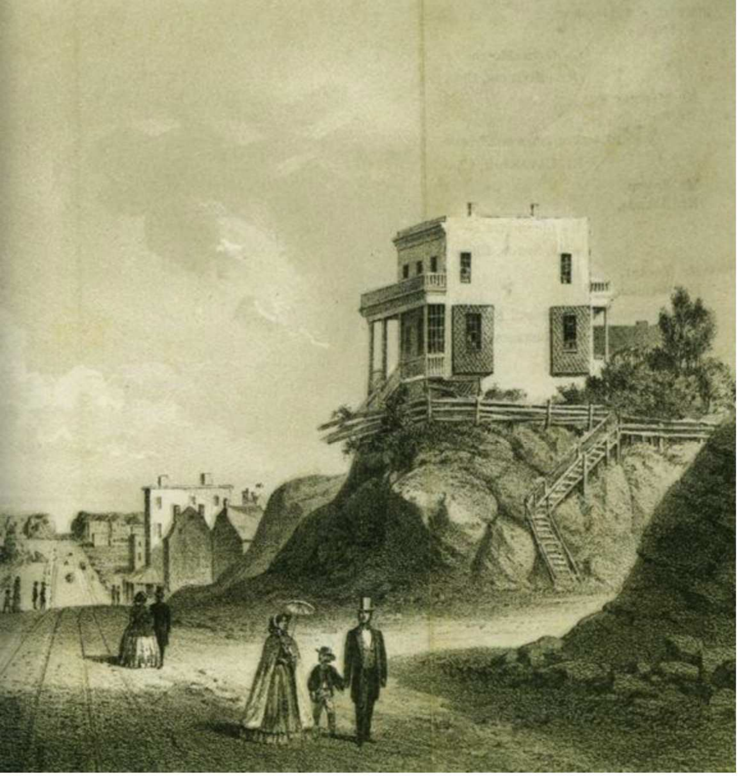
The British Headquarters map 1776

Towards the end of the American Revolution, this British map reveals the fundamentals of Manhattan, almost unchanged since 2 centuries ago, except a town of 32, 000 inhabitants at the bottom of the island (originally New Amsterdam, with its crooked and bent streets, which would eventually become the richest financial district in the world).



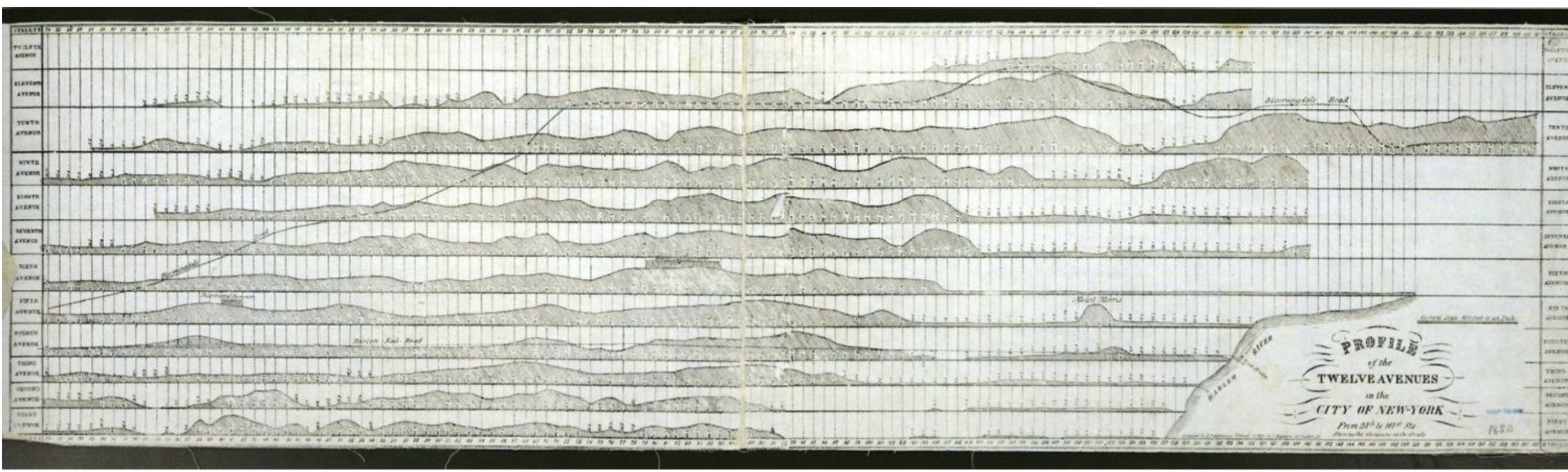
North of Manhattan, the low marshlands of Harlem



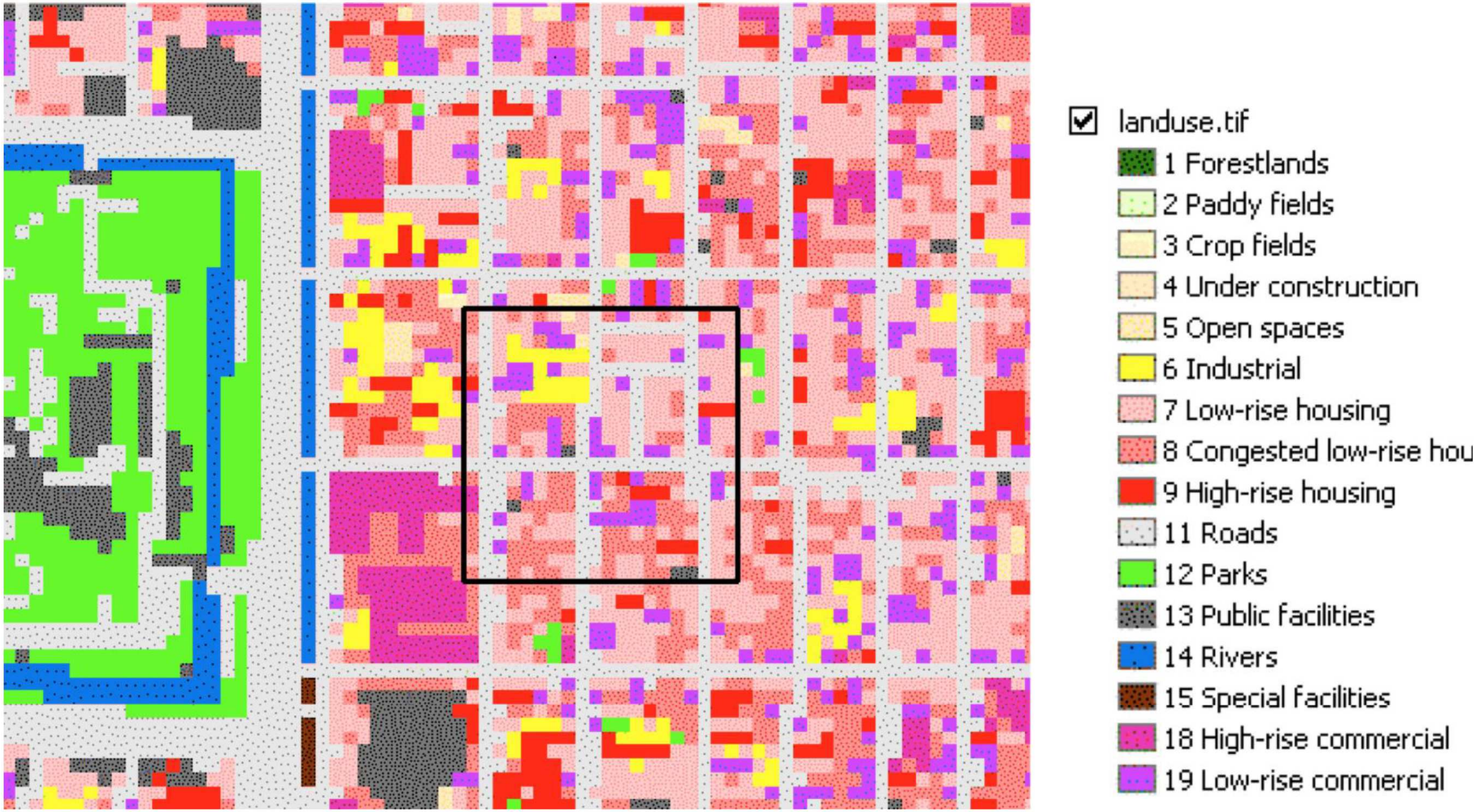


Below, Profile of the 12 avenues in 1850. Although the historical elevations along these profiles have been flattened to make way for the grid (see left house in the air in a print of 1861) the difference today is not as dramatic as one might suppose. The elevation profiles of Manhattan's avenues today show a remarkably similar parttern to that depicted in Hayward's map in mid 19th century.

Source: Hilary Ballon, *The Greatest Grid*



Layer 5: In contemporary cities fractal mix use has been replaced by zoning (Fine grain land use in Kyoto, the black central square is 200 m side)



Layer 6: The fractality of the 3D urban structure

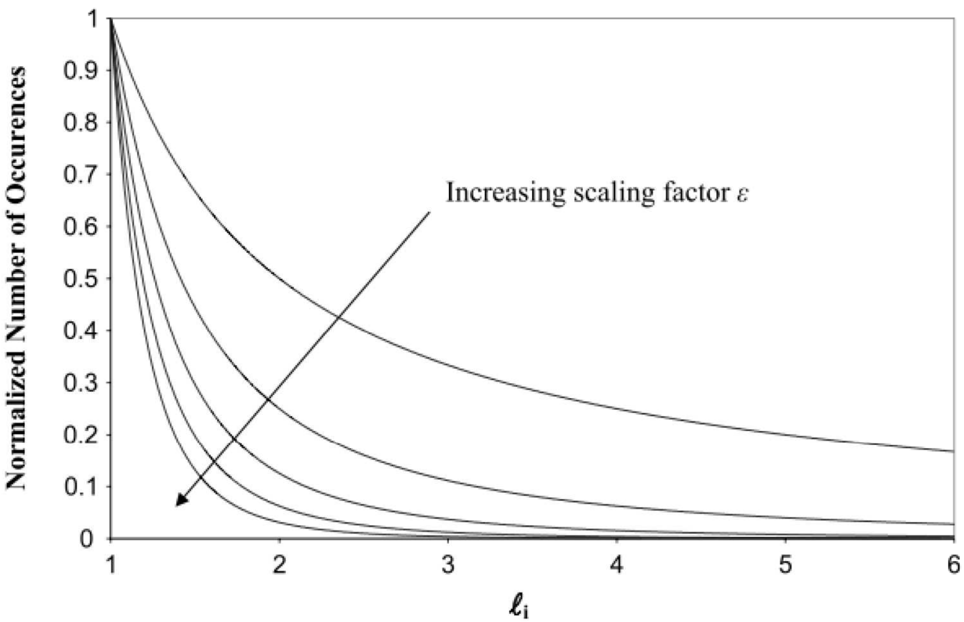
- Impacts on density and land prices through FAR
- Impacts on economic density
- Impacts on energy density
- Impacts on urban microclimate (rugosity and wind speeds, dispersion of pollutants, heat island effects, etc.)

Do average values mean anything in urban studies ?

Is the urban world Gaussian ?

Key result 1: The urban world is not Gaussian. It follows inverse power laws with extreme inequalities in intensities.

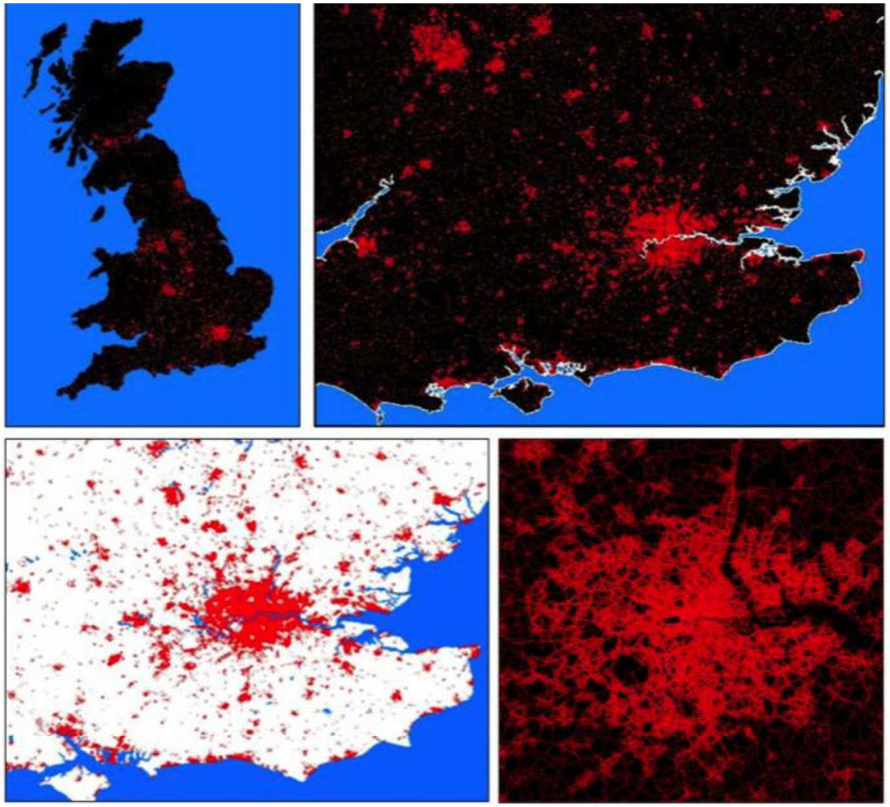
$$freq_i = \frac{A}{l_i^{\square}}$$



Relationship between scaling factor ε and normalized frequency distribution.

Power law scaling consists of universal properties that characterize collective phenomena that emerge from complex systems composed of many interacting units. Power law scaling has been observed not only in physical systems, but also in economic, financial and urban systems, shedding new light on economics, and, in recent years, has led to the establishment of a new scientific field bridging economics and physics.

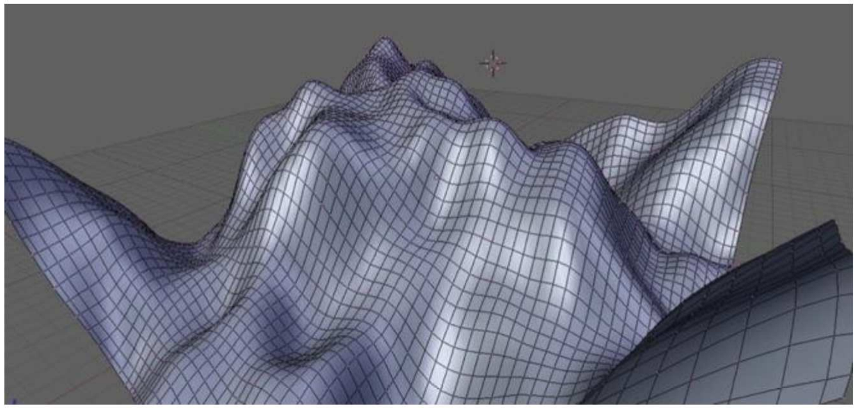
Key result 2: The multiscale problem of the urban world requires a multiscale multifractal approach

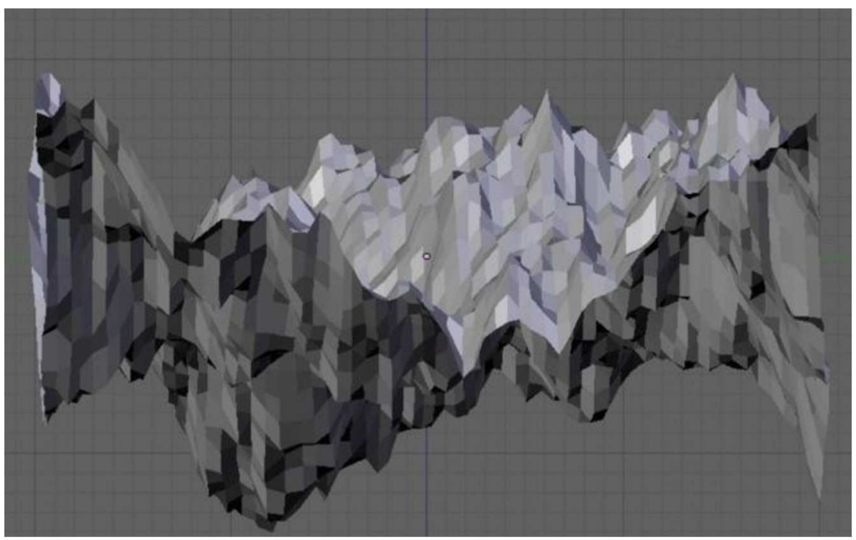


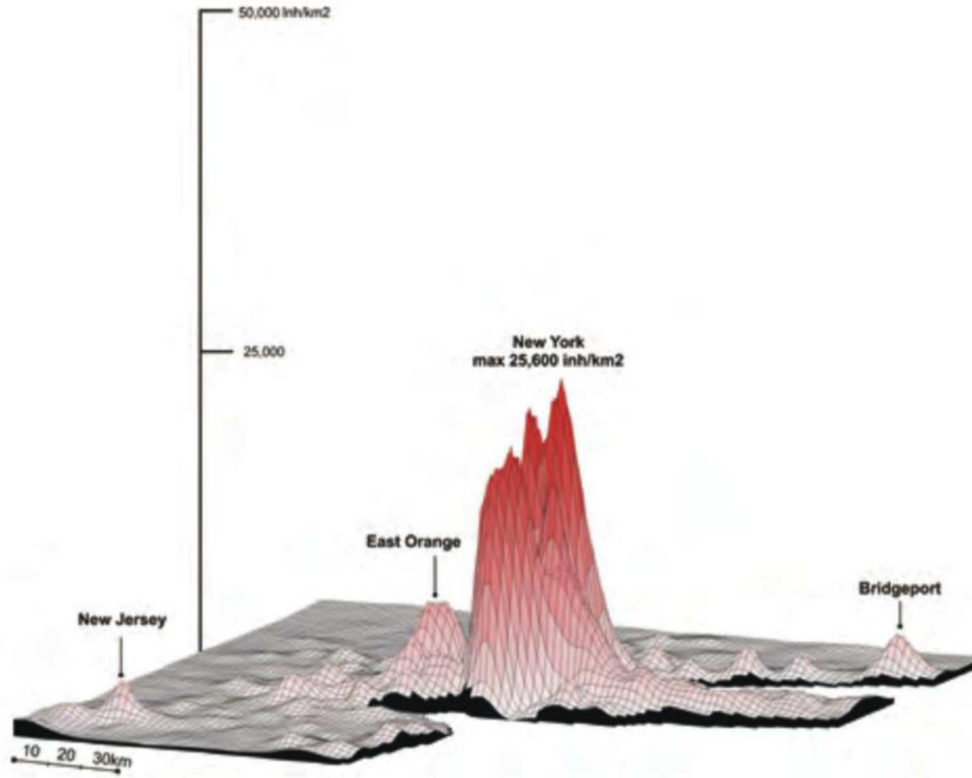
Self-Similar Urban Morphologies from Population, Remotely Sensed Imagery and Street Network Representations

Top left and right show the urban morphology of the UK and the South of England from 1991 gridded Population Census Data. Bottom left is an image taken from RS Data for 2000, and bottom right is from street network data for Greater London. Note the clusters on all scales that accord to the rank-size scaling

Source: Michael Batty, *The new Science of Cities*







Residential density in New York

Source: The World Bank

Cities evolving over long periods of time display scale-free structures, multi-connectivity and multifractal structures

– Scale-free structure

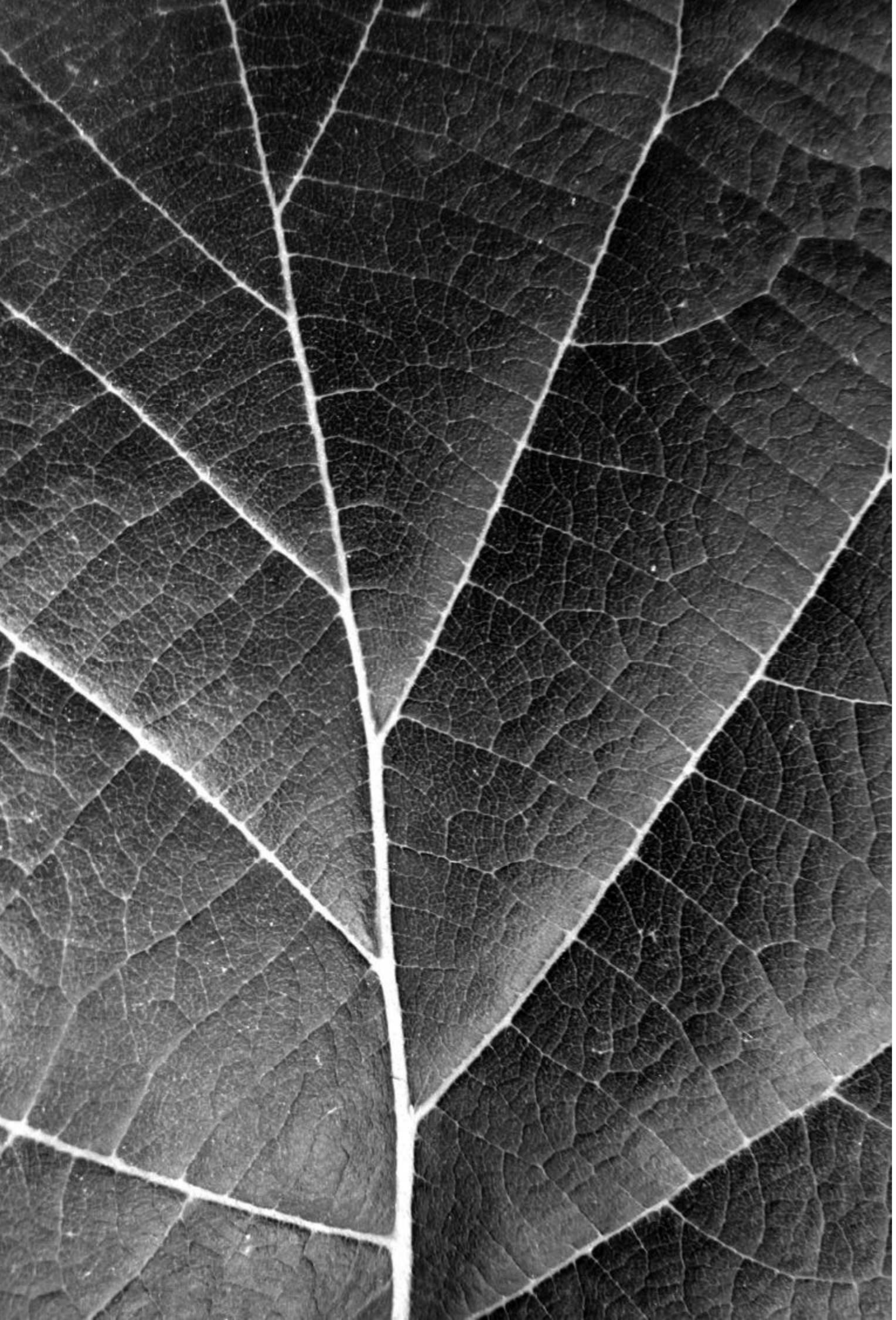
- The urban system is made of many subsystems at different scales
- The urban system displays a high diversity of sizes and scales
- The urban system is complex and diversified at every scale; it is scale-free

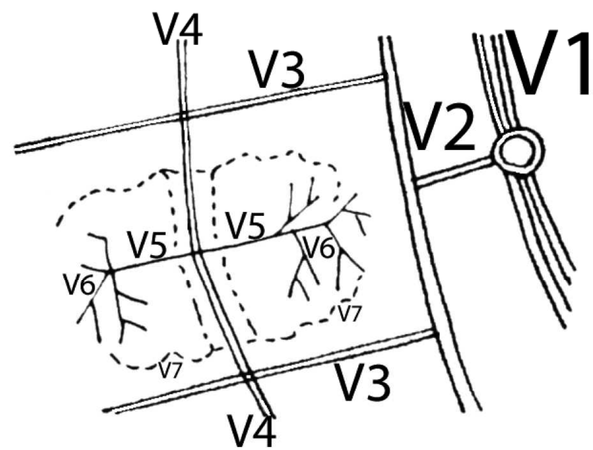
– Multi-connectivity

- The urban system is highly connected through a variety of short, medium and long range connections
- No constrained hierarchy: a sub system of a given size/scale can be directly connected to any sub system of any size/scale
- Connections display mathematical regularities described by graphs theory

- Multifractal structure

- The urban structure is the layering and interlocking of different fractal structures belonging to different morphological periods
- The urban structures fractal parameters vary locally



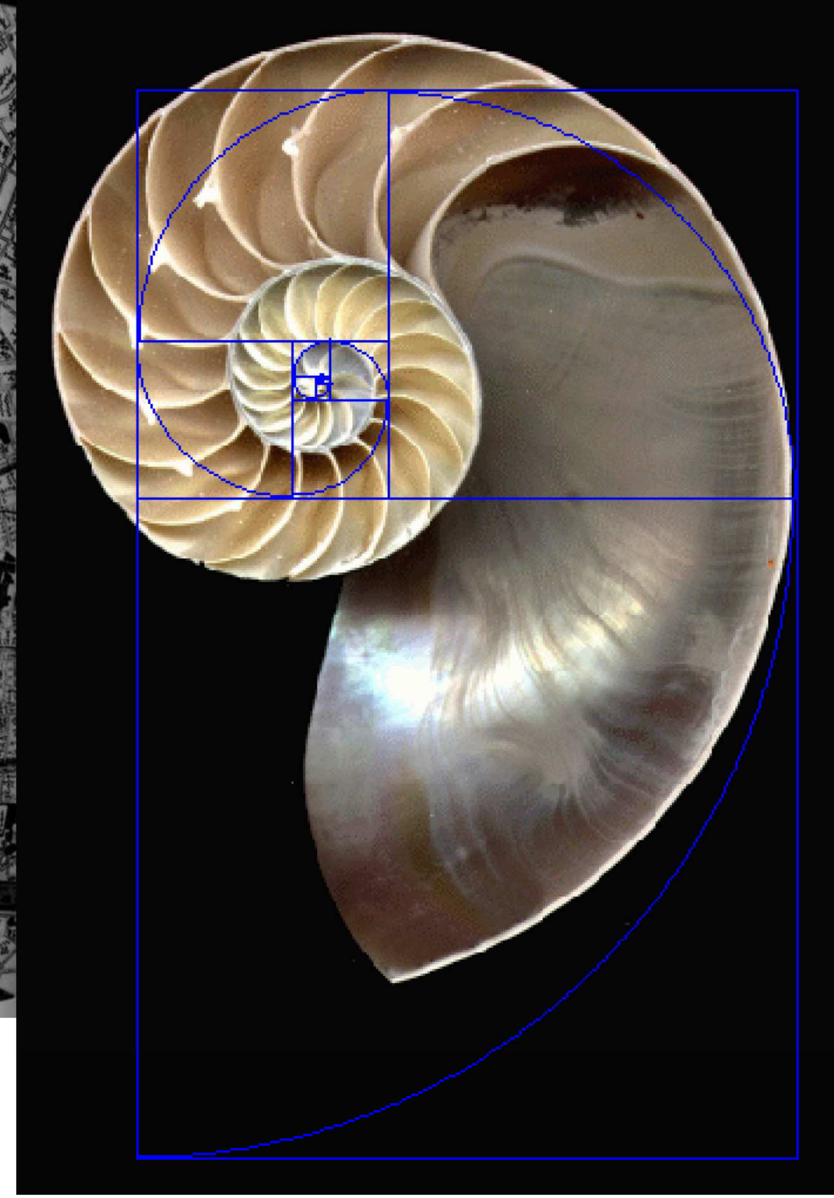


Multi connected leaf pattern
Multi connected urban pattern (top)
Versus disconnected constrained hierarchy street pattern (bottom: Le Corbusier's rule of 7 V) Source: Serge Salat, *Cities and Forms*

Source: Serge Salat, *Cities and Forms*

The Euclidean grid continually transformed into a multifractal labyrinth. Plan of Herat in Afghanistan.

Divided in two, the plan shows the coexistence of different urban geometries on different magnification scales. Like in Jaipur, the overall geometric pattern of streets is clearly a regular, large-scale cosmic diagram but the secondary network is much more irregular. These two networks can no longer be perceived on the scale of local connections where it gives way to a very irregular, fine-grained weave of triangular courtyards and dead ends. Far from being a simple opposition, the dialogue between the regular and the irregular is a dialectic between a regular pattern, the growth of irregularity, and its regularization (Rome at its foundation, medieval Rome, and Rome of Sixtus V, for example).



Edo (Tokyo) plan: a fractal spiral

Source: Serge Salat, *Cities and Forms*

Urban complexity emerges from breaks of symmetry

Cities are complex evolutionary systems far away from the equilibrium

1. **Urban systems follow the laws of far away from equilibrium thermodynamics (**Prigogine)

Cities do not automatically return to equilibrium for they are forever changing, indeed they are far-from-equilibrium. Dissipative structures far away from equilibrium increase the complexity of their structures to dissipate efficiently flows of energy.

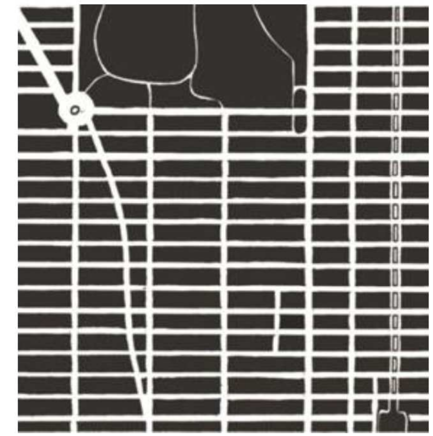
2. **Urban systems display emerging properties**

Dynamic processes build on existing patterns to reinforce size and to generate economies of scale. Patterns are largely built from modules operating from the actions of individuals (or at least individuals acting for groups and institutions) from the bottom up at relatively small scales. They evolve through time in such a manner that any snapshot at any cross section shows an emergent order that is the product of countless decisions.

3. **Urban systems have been selected by evolution**

Resilient cities that have a long history show the characteristics of complexity of evolutionary systems. They are the survivors of a process of creation/elimination analogous to the evolution of life.

Urban complexity is an emergent property fostered by breaks of symmetry in the urban fabric

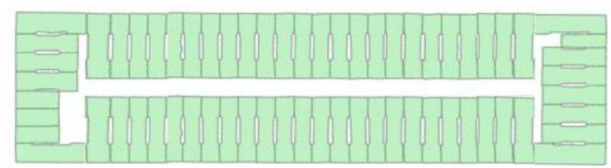


Fine grain

- Fine grain street network
- Fine grain platting
- Fine grain mixed use and diversity

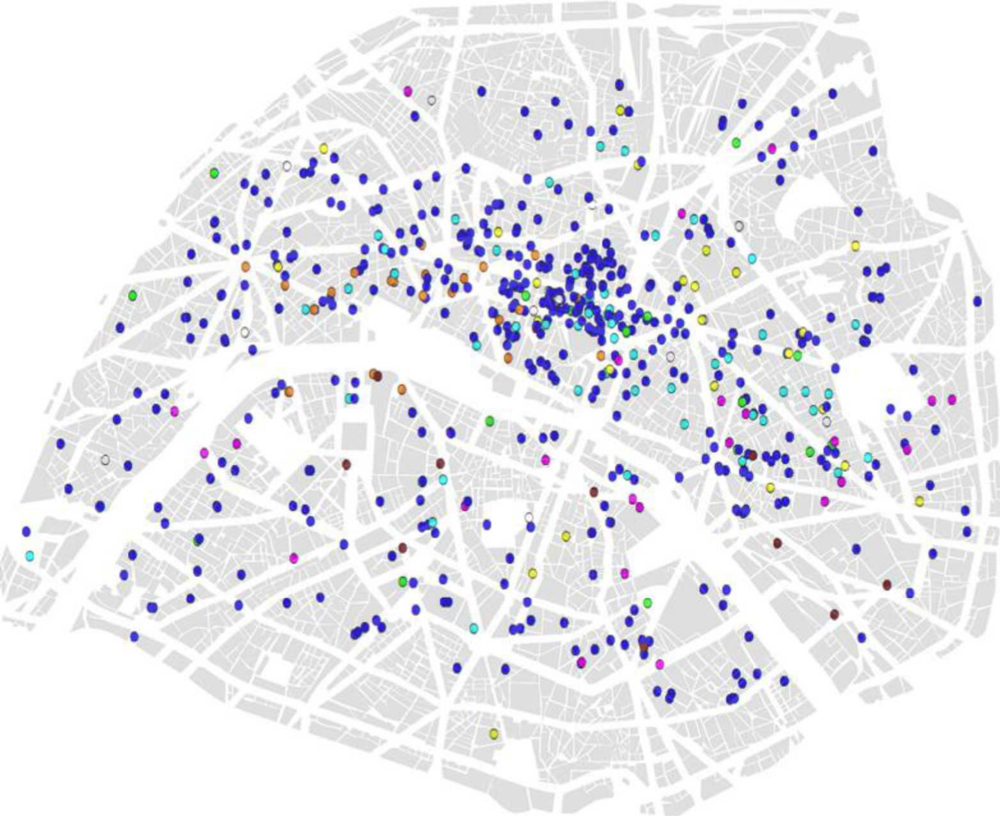
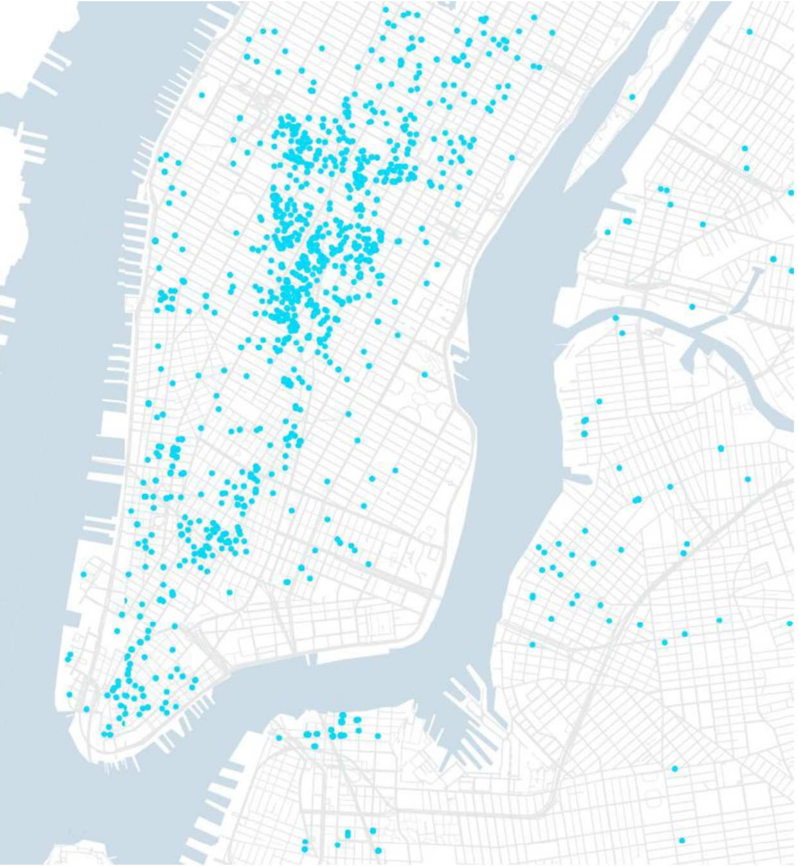
Breaks of symmetry in urban space

- Street width and properties (avenues in Manhattan are 13 times more continuous and connective than streets in terms of graphs theory)
- Elongated block size
- Plot size



Breaks of symmetry and fine grain must be embedded in the master plan to support urban resilience and allow complexity to increase.

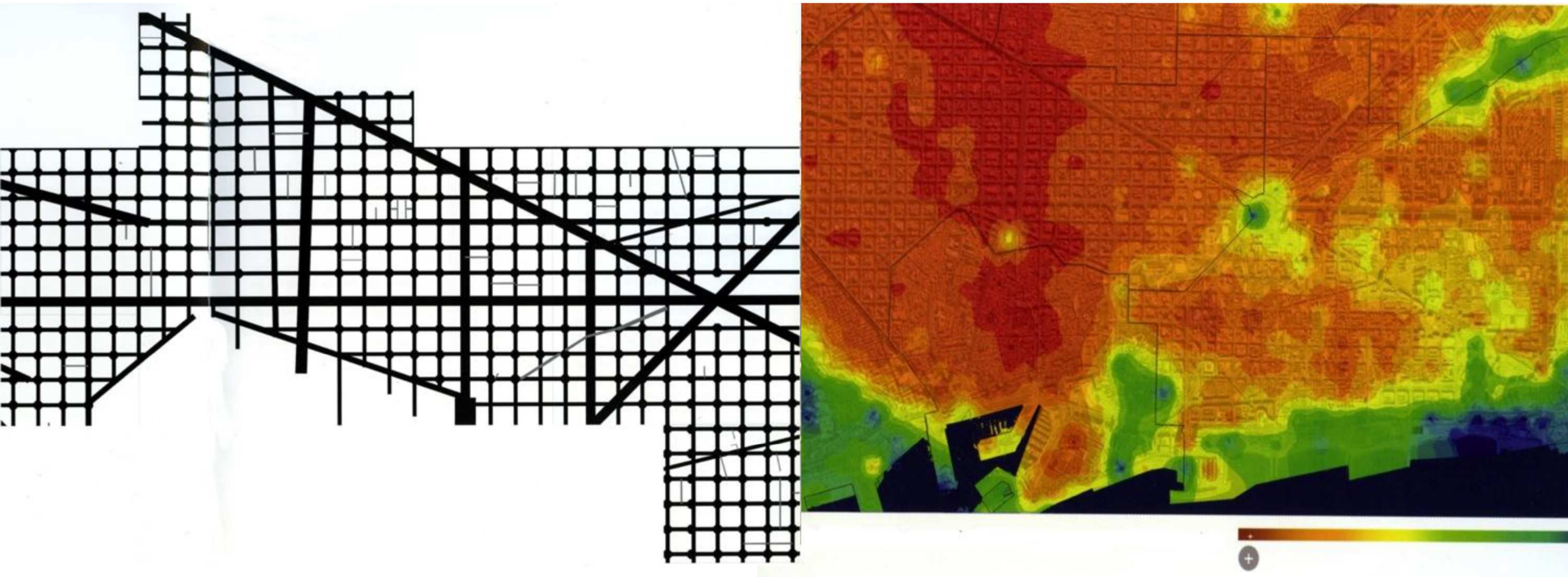
Source: Urban Morphology Institute

Digital economy in Paris and Manhattan (note the impact of Broadway)

Sources: Left: Urban Morphology Institute / Right: Andrew Laing CURE

Breaks of symmetry within the urban grid foster the emergence of economic structures in Barcelona



The diagonal avenues don’t break the symmetry. Passeig de Gracia does (spatially and historically). Around Passeig de Gracia the economic activity concentrates (measured here by a Shannon index of diversity of the legal entities)

Source: Agencia Ecologica de Barcelona

The multifractal structure of the urban landscape has strong macro and micro-economic impacts and energy impacts at all scales

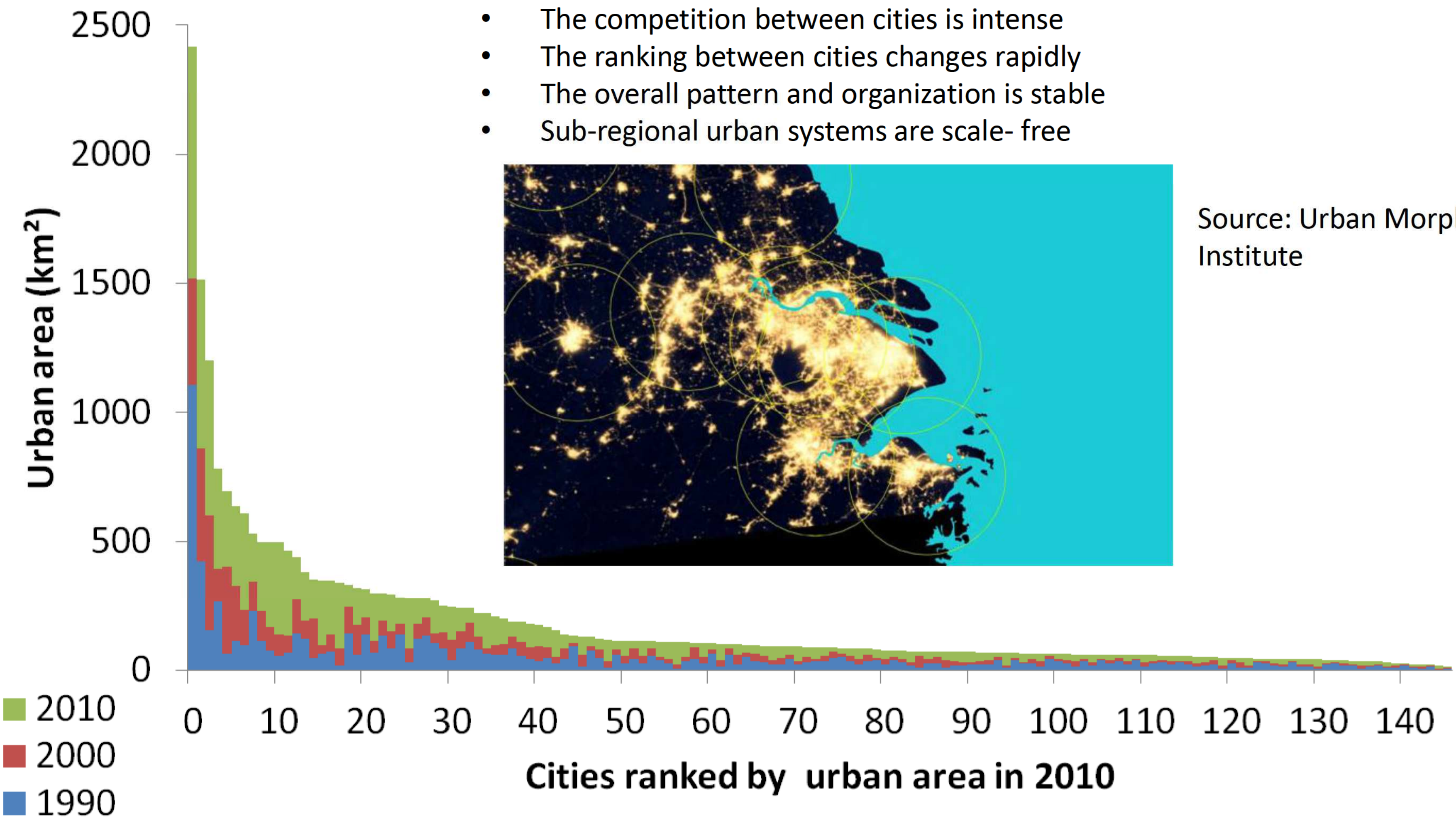
Hierarchy in urban systems and inverse power laws

- The relationship between cities in a hierarchy of central places is one of scaling in that the typical rank-size distribution is represented as $P(r) \sim r^{-1}$. This is the pure Zipf (1949) relation, first popularised in his book *Human Behavior and the Principle of Least Effort*.
- In reality the hierarchy exponent varies between 0.7 (US, Japan, Europe, European countries, central regions of China) and 1 or more (Korea, Eastern regions and sub-regions of China).
- In fact although rank-size scaling is highly stable through time, changes in the population of cities that make up such scaling can be highly volatile, and this remains a major puzzle in reconciling aggregate with disaggregate space-time correlations (Batty, 2010).

China’s urban system is a stable and integrated system of regional sub-systems

China’s urban system is resilient over time and keeps the same type of scaling structure.

- The competition between cities is intense
- The ranking between cities changes rapidly
- The overall pattern and organization is stable
- Sub-regional urban systems are scale- free

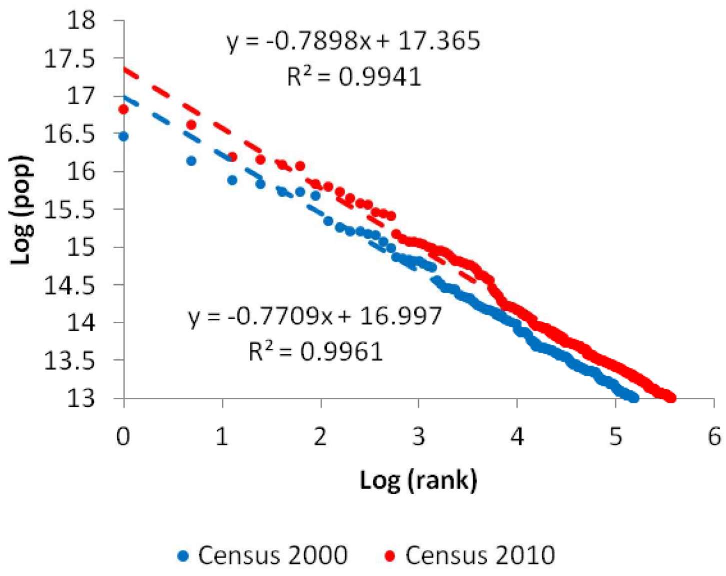


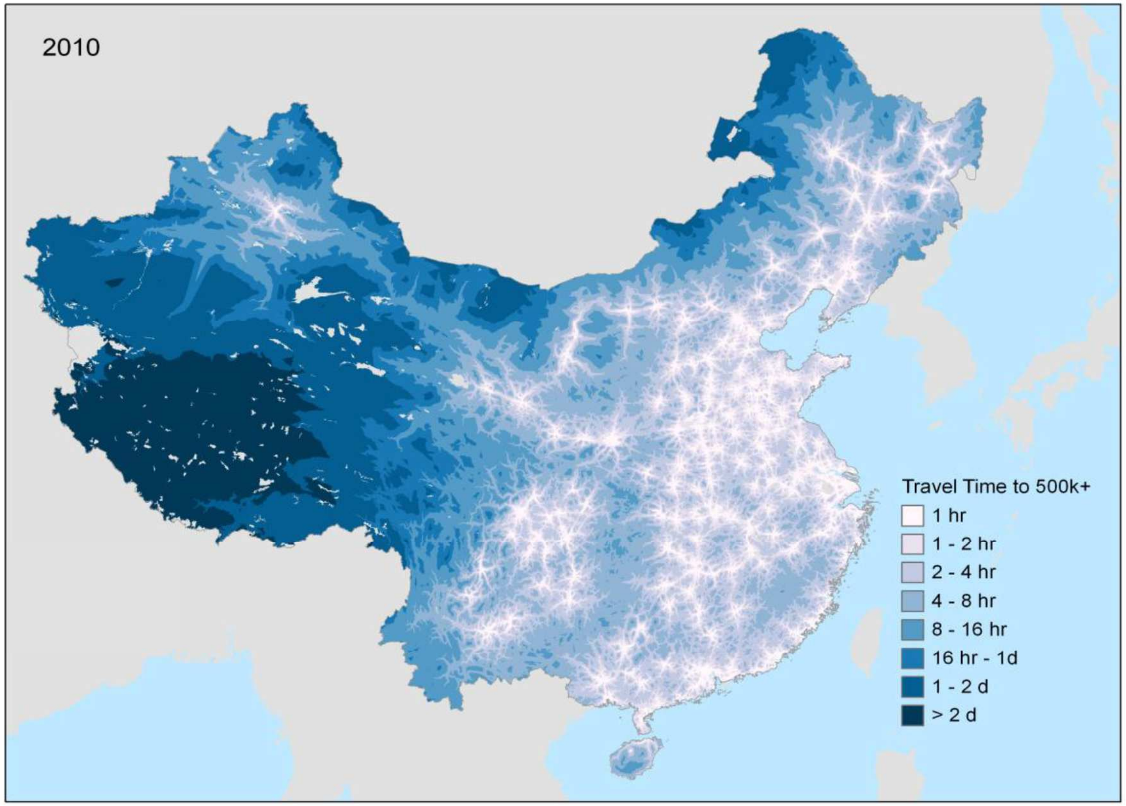
Source: Urban Morphology Institute

The organization of the Chinese urban system is now comparable to the US, Europe or Japan

- The ambitious inter-city transport plan (high speed rail and highways) has contributed to a nation-wide integration
- The governmental support to small and mid-size cities growth has been beneficial
- The hierarchy of China's urban system is now comparable to the US, Europe or Japan.

Source: Urban Morphology Institute and The World Bank (map)

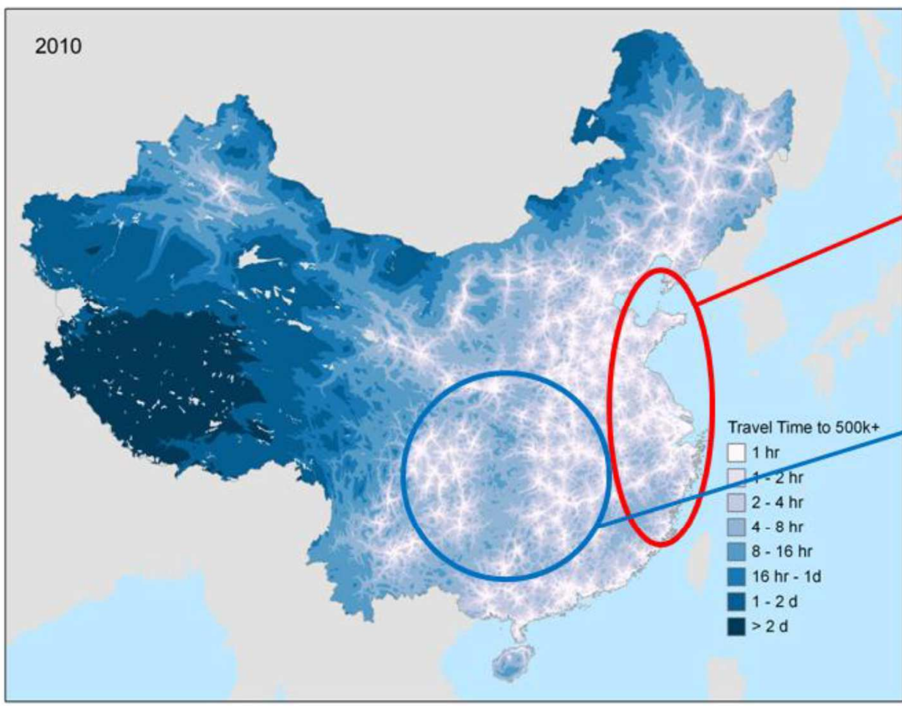


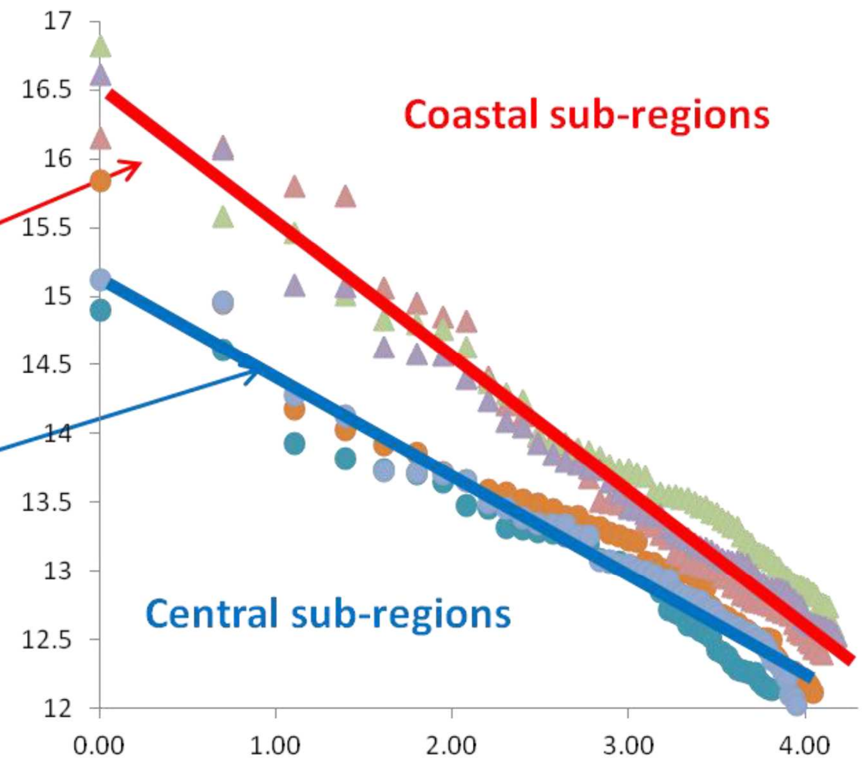


But there is a risk of over-concentration in coastal sub-regions

- In coastal regions, urban systems over-concentrate, and follow the South-Korean pattern
- In a huge country like China, this could lead to the emergence of too large cities (50 to 75 million inhab), with:
    - Diseconomies of scale in terms of energy
    - Negative externalities such as congestion, pollution and social inequalities

The over-concentration into massive megalopolis is likely to jeopardize the resilience of the entire urban system.



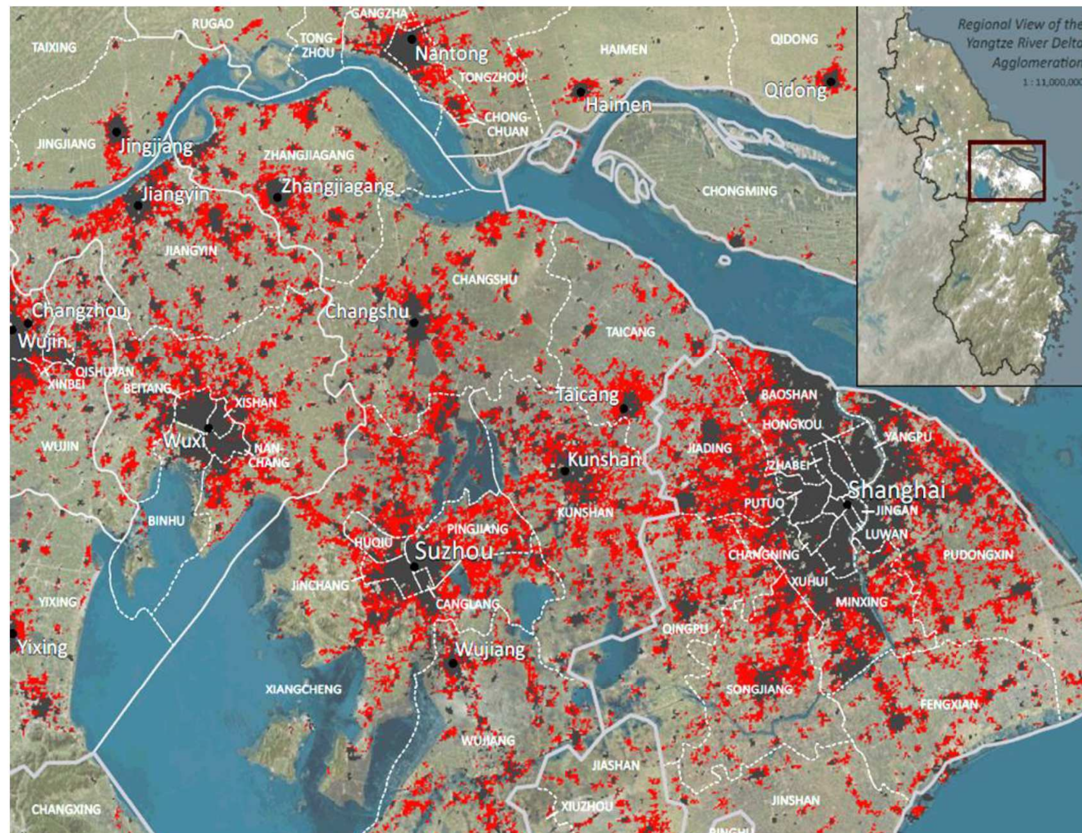


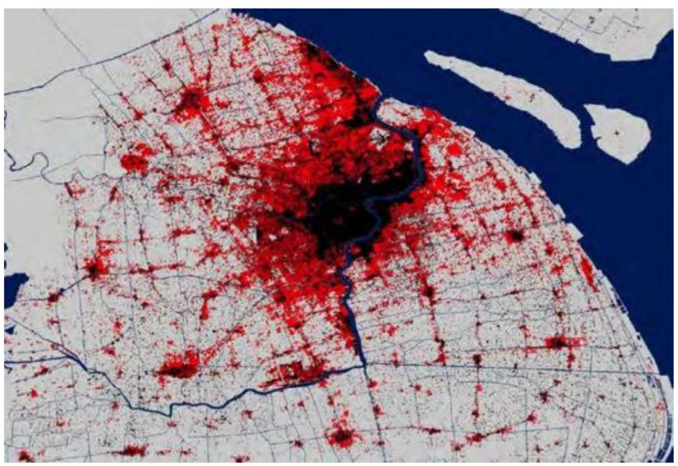
Source: Urban Morphology Institute and The World Bank (map)

Sprawl blurs the fractal spatial structure of urban systems
Density has been divided by 2 in 20 years in Chinese cities

- Massive conversion of urban to rural land feeds urban sprawl
- **Built up areas have been multiplied by 3.3 in 20 years**, much faster than population growth

Shanghai 2000-2010



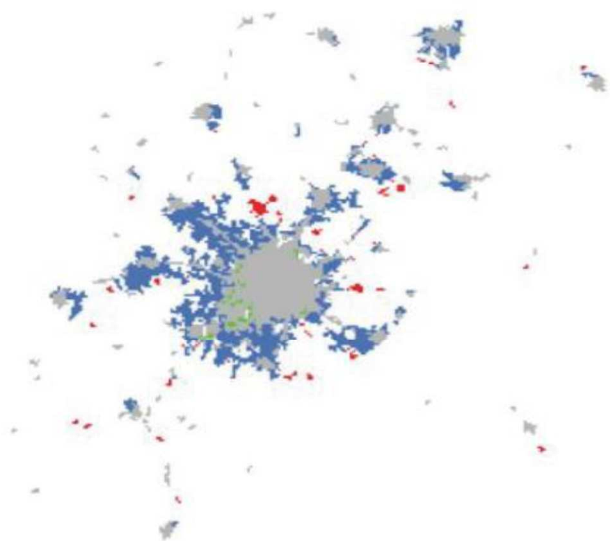


Shanghai 1990- 2000

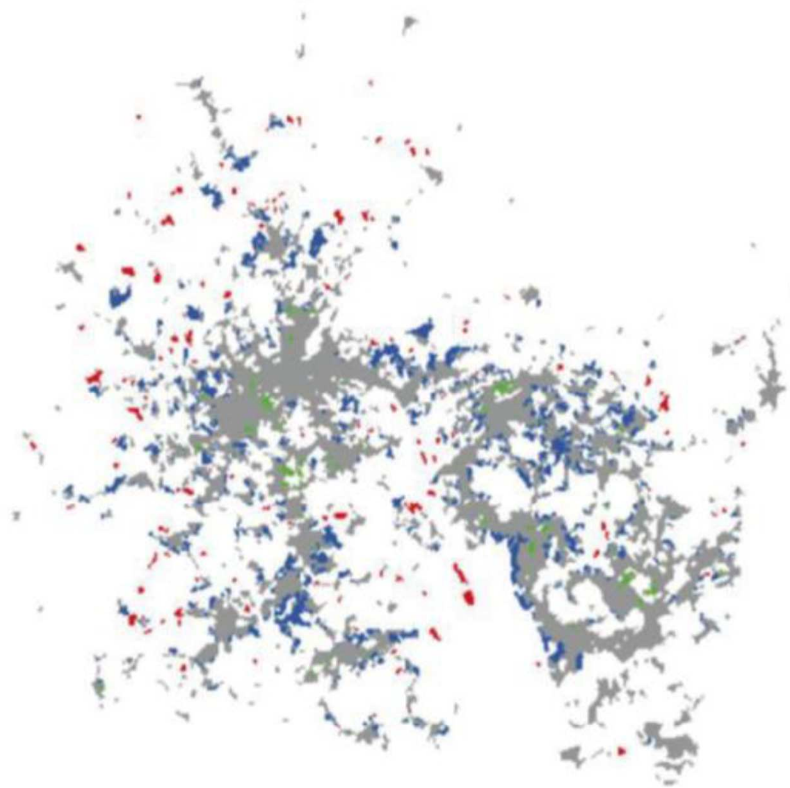
Map produced by University of Wisconsin-Madison, May 2013.
Administrative boundaries from University of Michigan – China Data Center.

Fragmented spatial expansion creates large scale fractals at the bigger scale with a disappearance of the smaller scales

In Chengdu and Guangzhou edge and leapfrog growth in the last 10 years represent 97% and 93% of urban spatial expansion, respectively. Low fractal dimensions are a measurement of loss of compactness



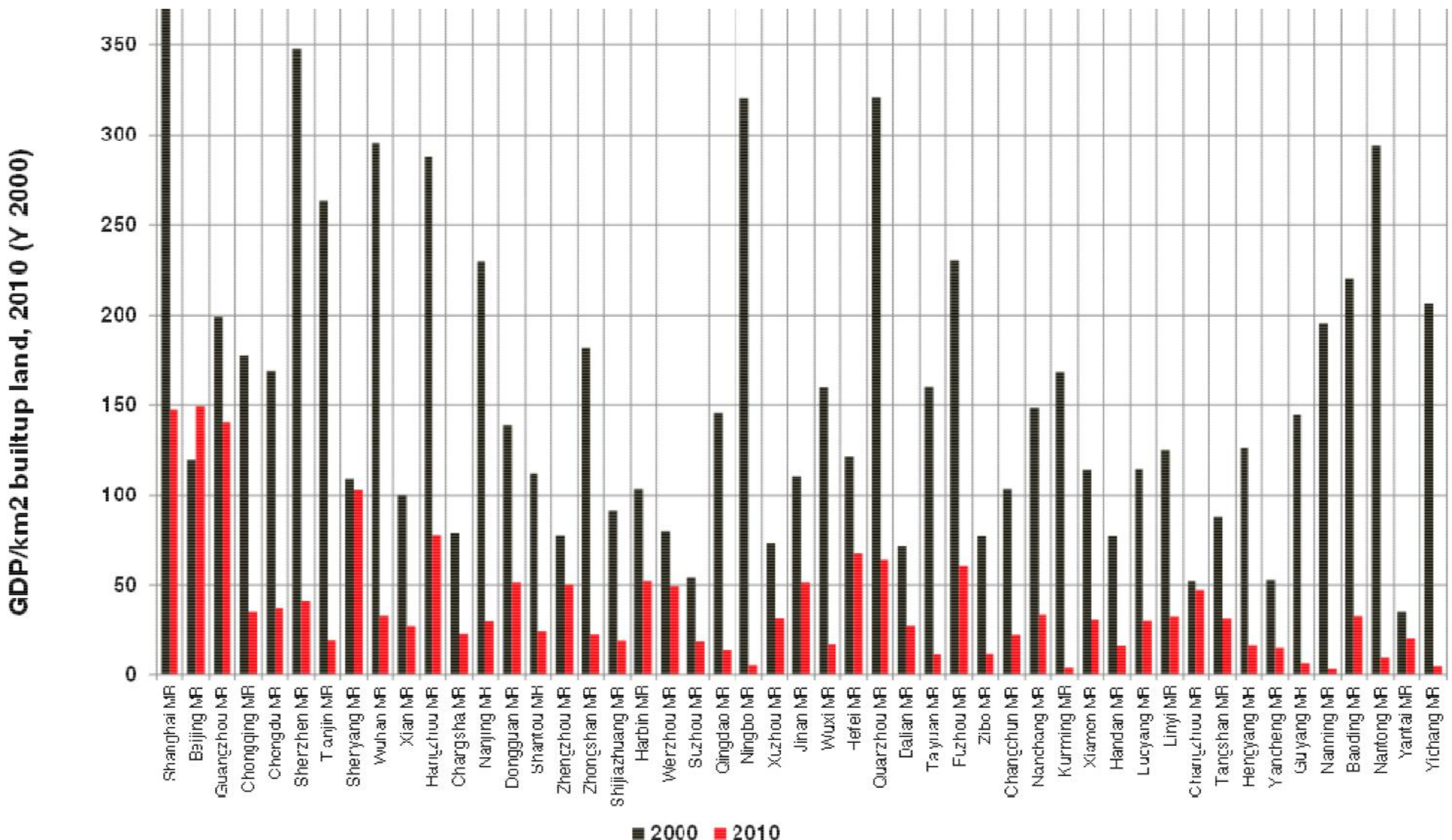
Chengdu
成都



Guangzhou
广州

Source: City Form Lab

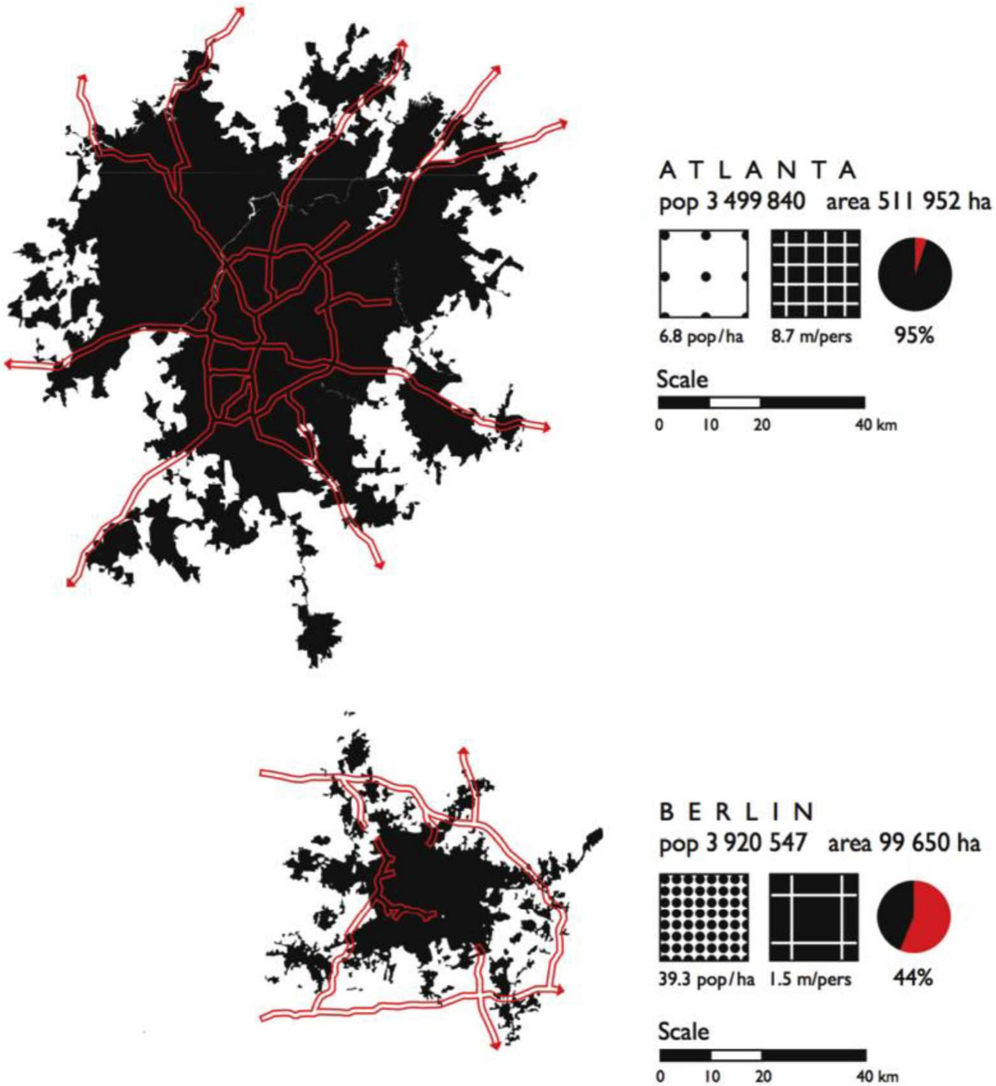
Marginal productivity of land use has fallen dramatically in almost all Chinese cities between 2000 and 2010



Source: Chreod 20113

Additional GDP/Additional km² in Shenzhen | 深圳 divided by 10
Additional GDP/Additional km² in Shanghai | 上海 divided by 2.5

Fragmentation and low density induces high per capita infrastructure costs and car dependency



With a similar population, Atlanta is 6 times less dense than Berlin

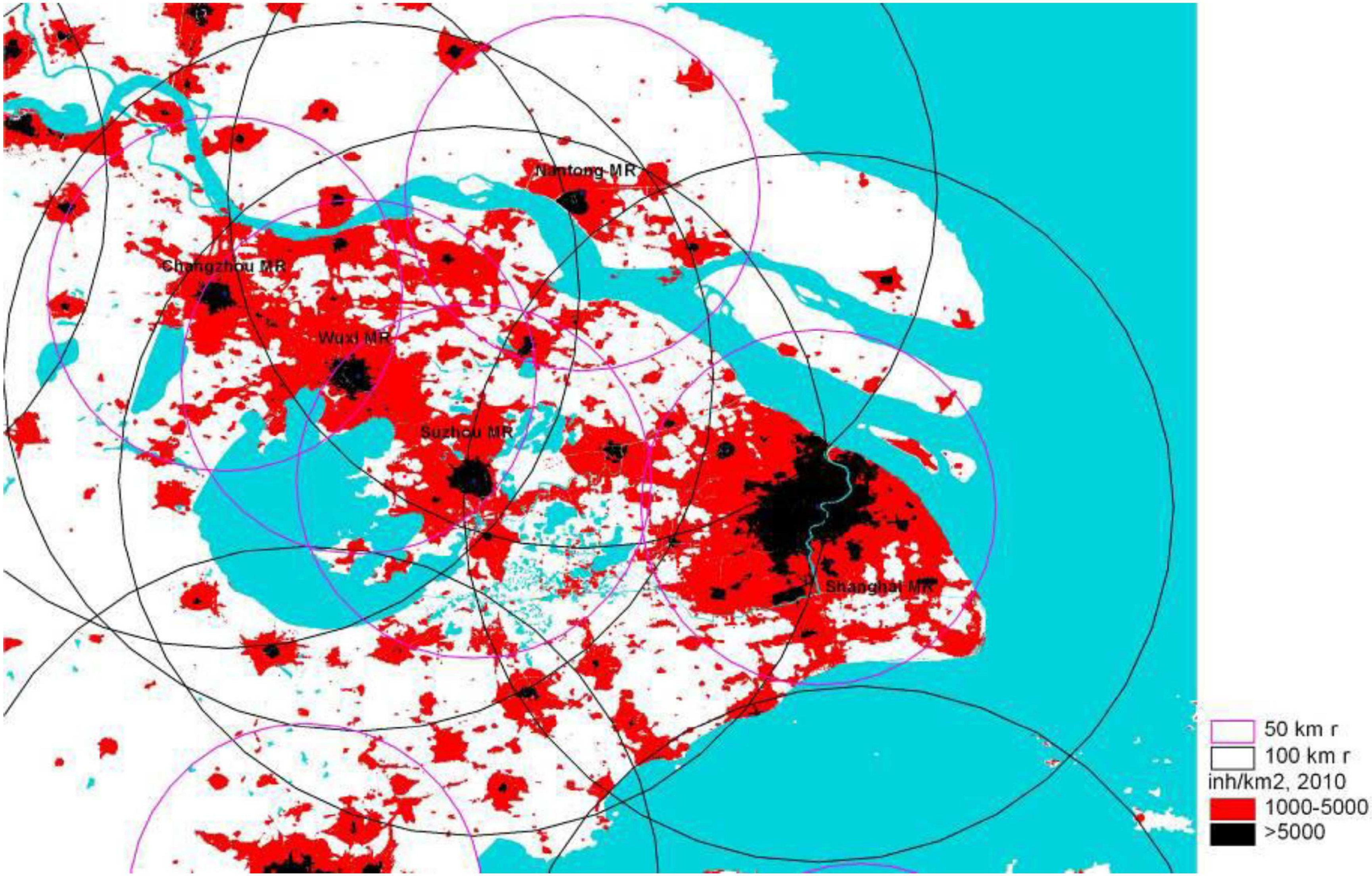
Infrastructure costs are 6 times higher in Atlanta than in Berlin

95% of people use a car in Atlanta, 44% in Berlin

Source: The World Bank

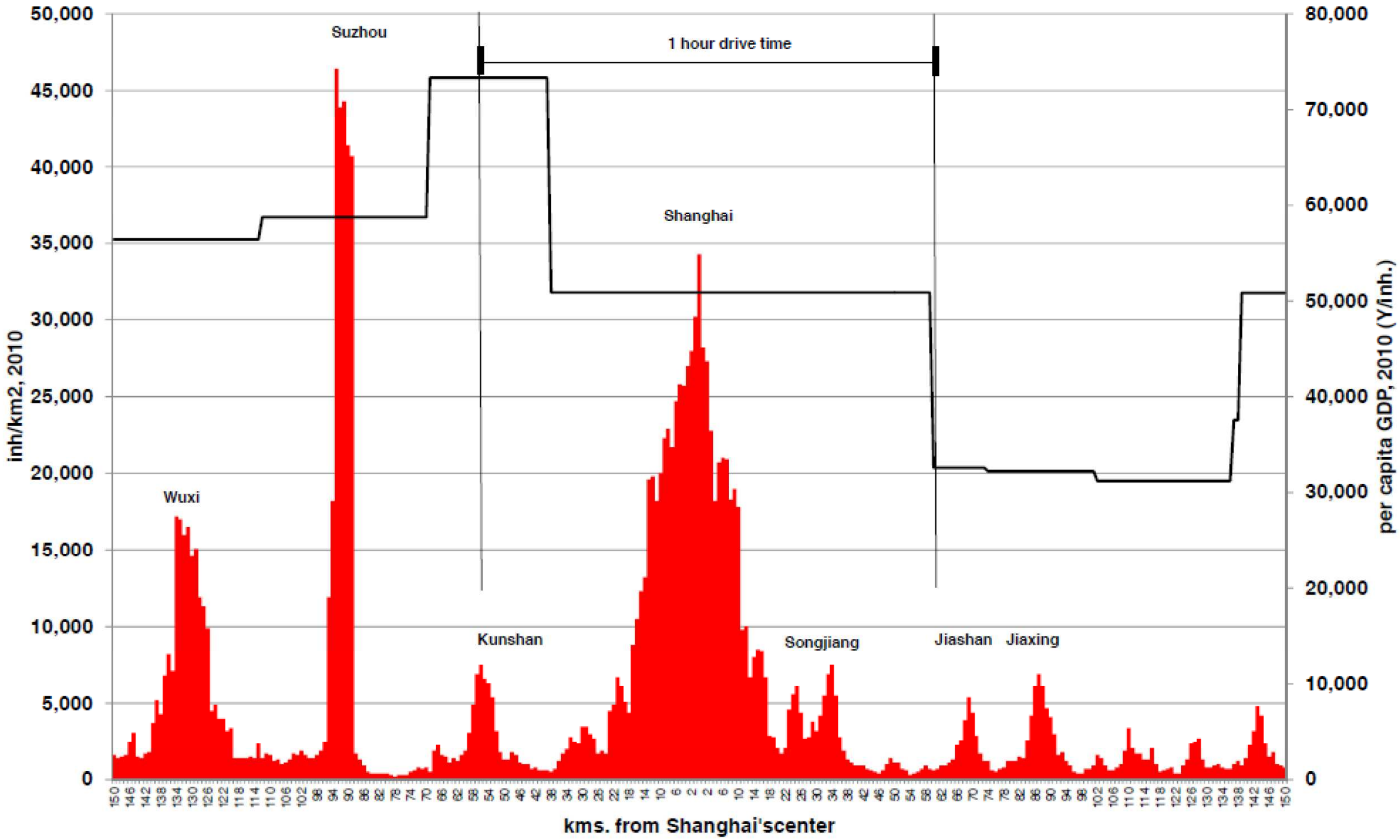
The demographic density remains unevenly distributed

Population Density, Yangtze Delta Region, 2010



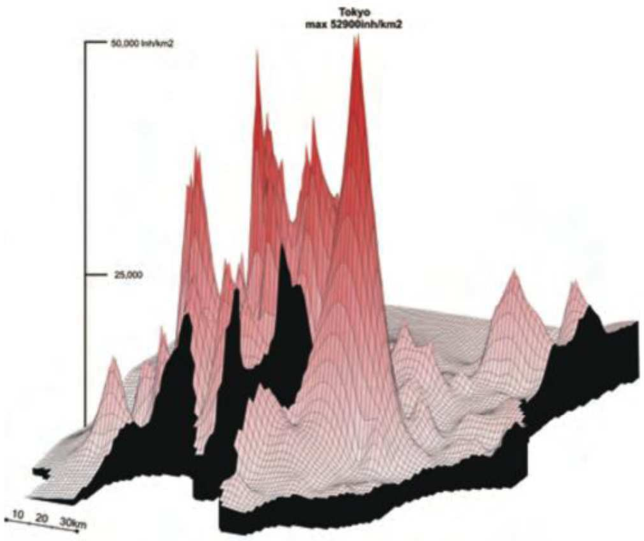
Source: Chreod 2013

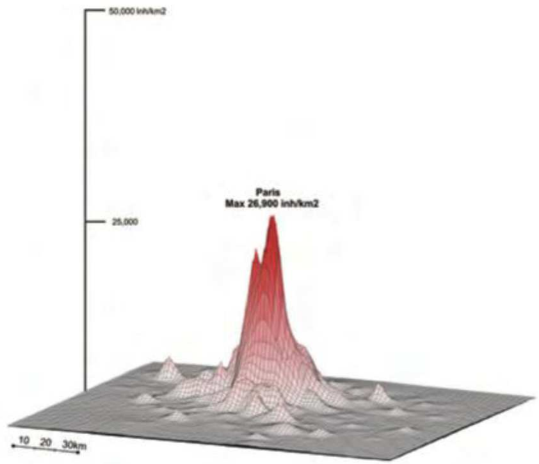
The unequal distribution of density leads to agglomeration Spillovers from Shanghai, 2010

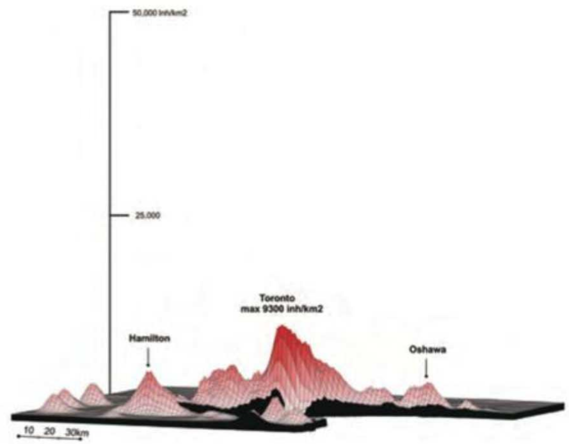


Source: Chreod 2013

Multifractal density with high variations reduces the emissions per capita and per unit of GDP







Tokyo
4.9 tCO2e/cap
146 ktCO2e/US$bn

Paris
5.2 tCO2e/cap
112 ktCO2e/US$bn

Toronto
11.6 tCO2e/cap
286 ktCO2e/US$bn

Beijing
10.1 tCO2e/cap
1,107 ktCO2e/US$bn

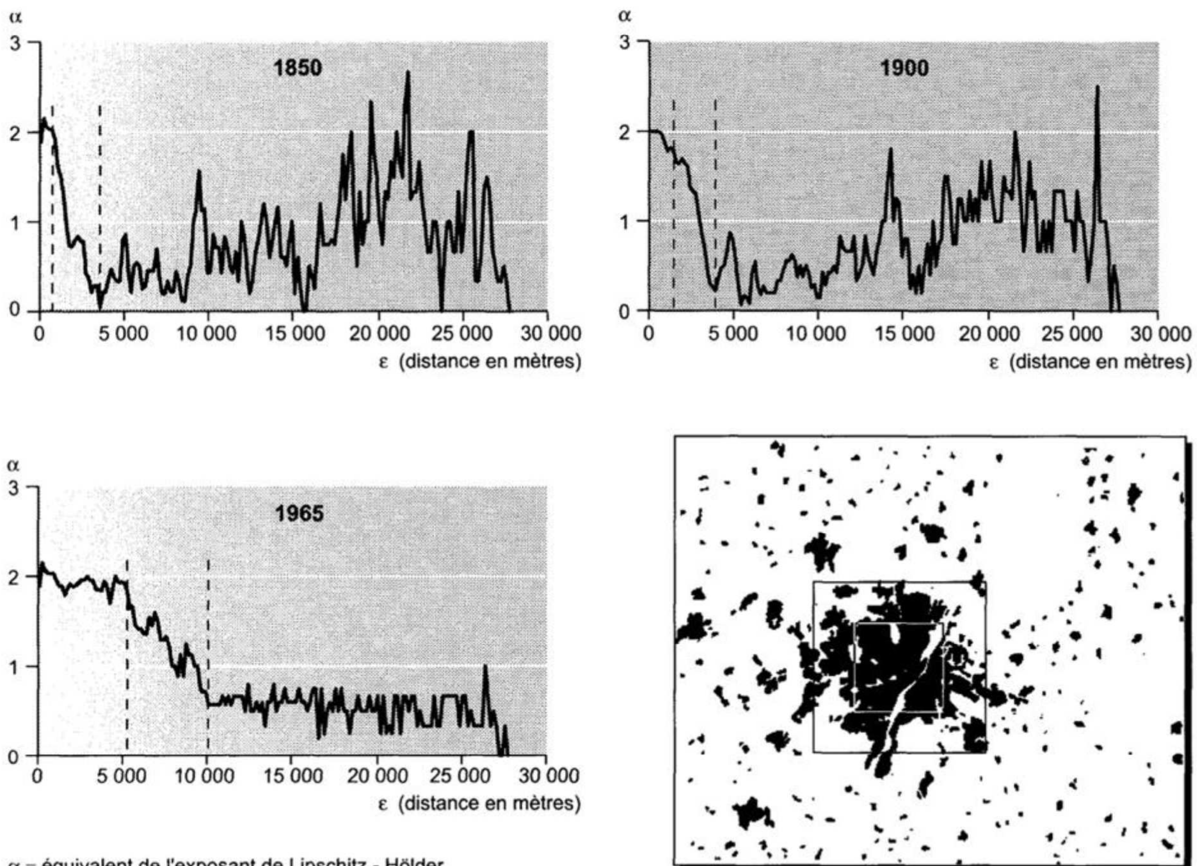
Shanghai
11.7 tCO2e/cap
1,063 ktCO2e/US$bn

Tianjin
11.1 tCO2e/cap
2,316 ktCO2e/US$bn

Source: The World Bank

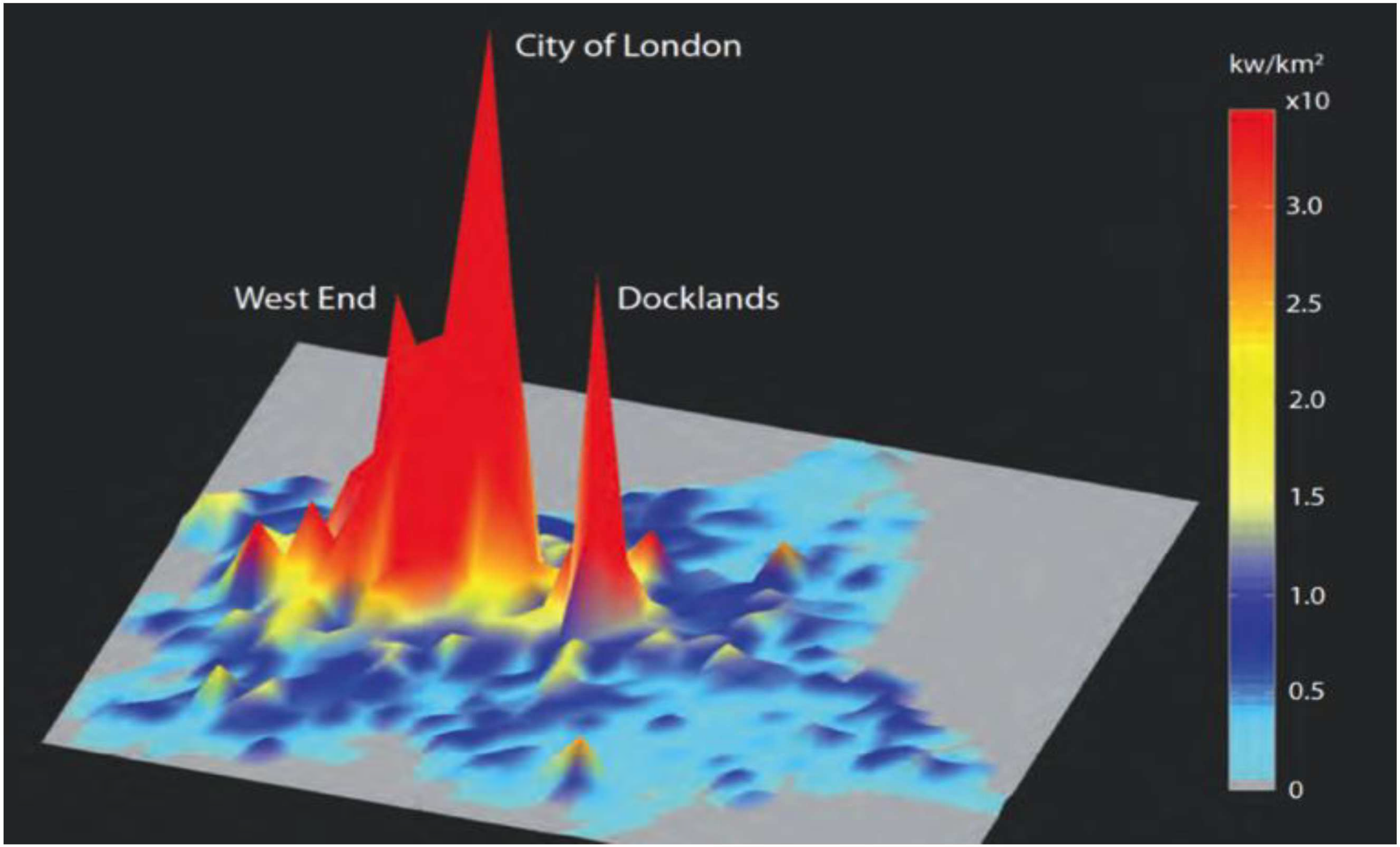
Multifractal analysis of metropolises shows the loss of multipolarity overtime with sprawl and spatial expansion

La courbe du comportement scalant de l'agglomération de Munich pour différentes périodes



Source: Pierre Frankhauser

In London, peaks of energy coincide with peaks of economic value creation On one sq mile, City of London is built within the pre 1666 Great Fire street pattern



Cities and systems of cities are

multiscale and multifractal structures

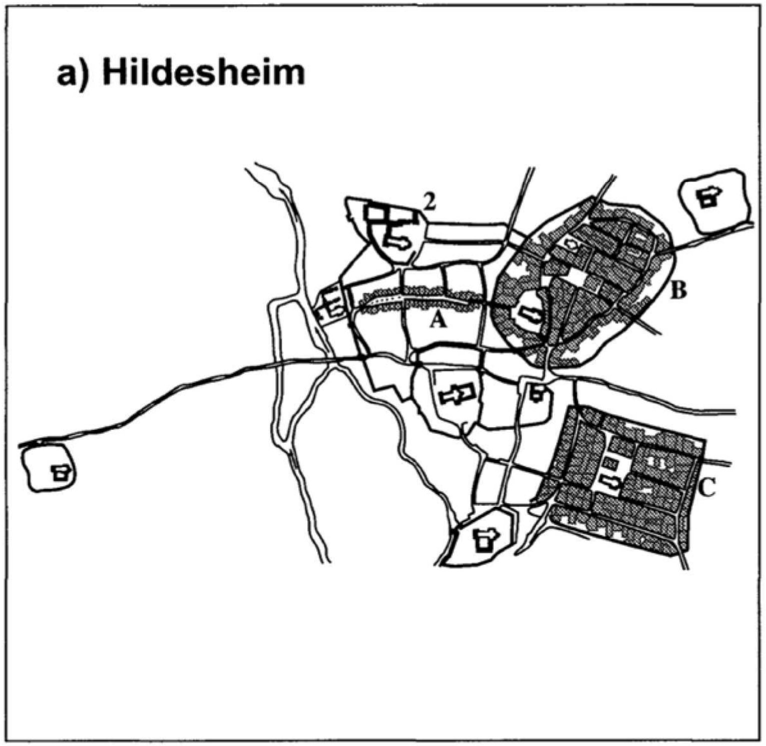
emerging from a myriad of interactions over long periods of time and shaped by intense political, social, economic competition

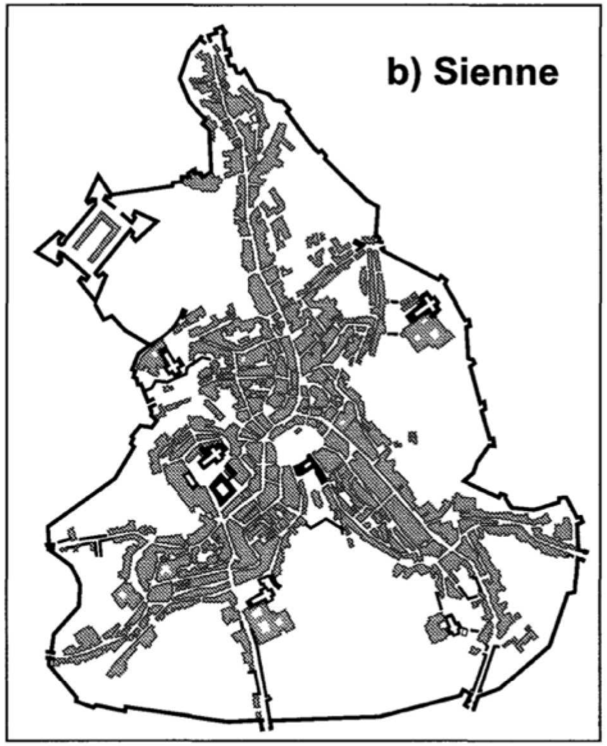
The urban landscape is a complex layered structure with alternances of destructions and expansions

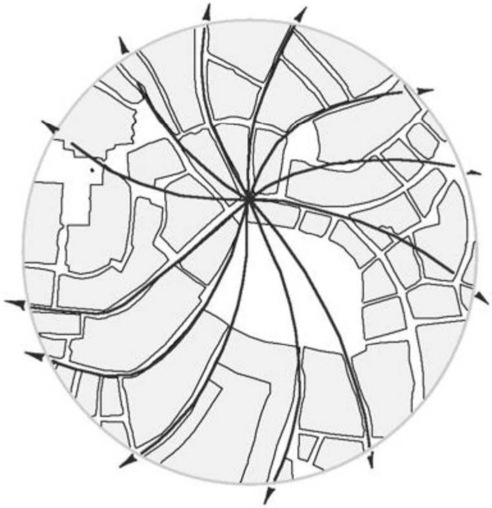
- Multi-cellular cities
- Palimpsest cities

Multi- cellular cities

Les tissus bâtis de Hildesheim et de Sienne



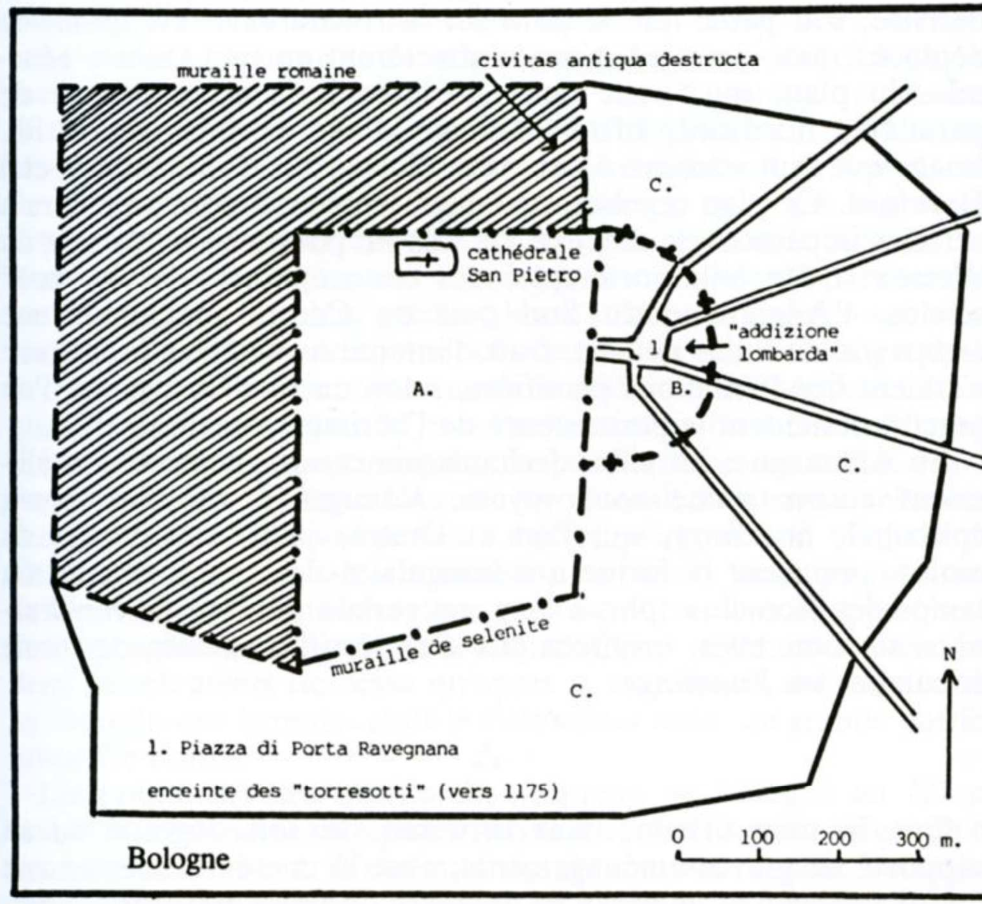




À gauche, le tissu de la ville de Hildesheim au 13e siècle. On distingue un bourg primitif *A* à proximité du siège épiscopal carolingien et de la cathédrale (**2**). À ce bourg se sont ajoutées deux cités (*B*, 12e siècle et *C*, 13e siècle). Plus tard, une enceinte commune entourera l'ensemble des habitats et des édifices religieux. À droite, le plan de Sienne vers la fin du Moyen Âge. Le tissu bâti paraît assez irrégulier. On distingue une croissance tentaculaire le long des rues principales. En réalité, la ville est aussi constituée de plusieurs bourgs, à l'origine assez éloignés les uns des autres (Braunfels, 1977).

Bologna

Source: Serge Salat, *Cities and Forms*



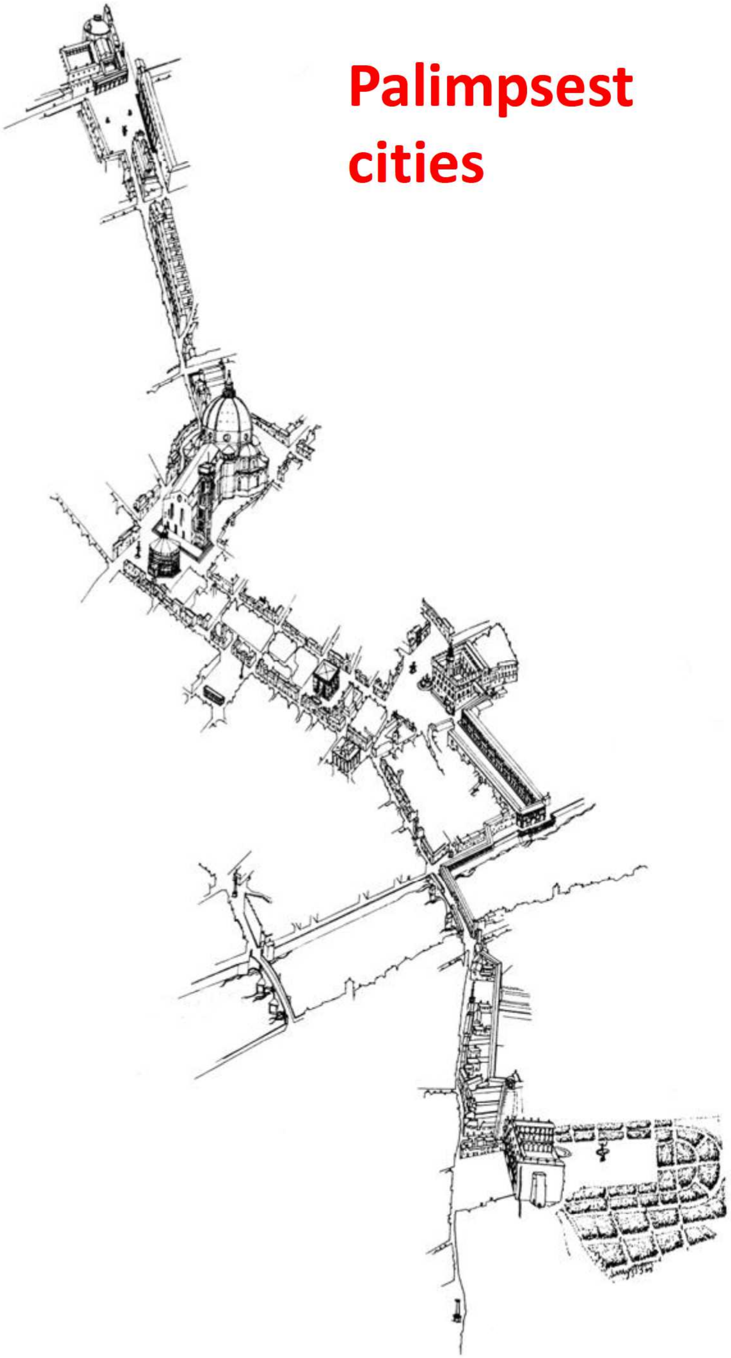
Bologne : contraction puis expansion.

Repli d'abord dans un quadrilatère qui occupe à peine la moitié de l'ancienne *civitas* et où un groupe épiscopal s'installe à l'extrême limite nord, au long des murs. En dehors de ce réduit, une zone dévastée est laissée à l'abandon, mal drainée, et devient insalubre. À cette première « ville barbare » (A), les Lombards adjoignent un camp militaire plus tard urbanisé (B) dont le dessin semi-circulaire et les axes disposés en éventail commandent ensuite l'expansion de l'habitat dans la partie est de la ville enclose à la fin du XII[e] siècle (C).

Source: Serge Salat, *Cities and Forms*

Firenze: From 200 towers fighting each other and inner permanent civil war in the 13th century to the Medicean order: multifractality emerges out of a Roman grid

Source: Serge Salat, *Cities and Forms*



Palimpsest cities

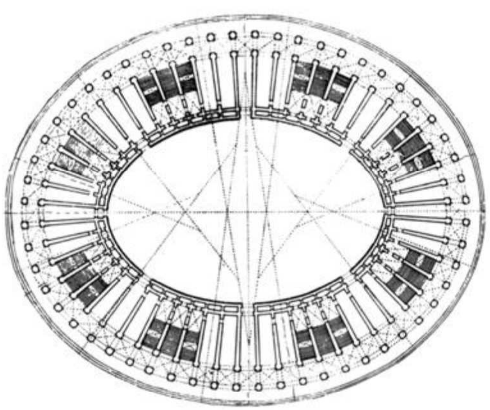
Firenze near Piazza Santa Croce

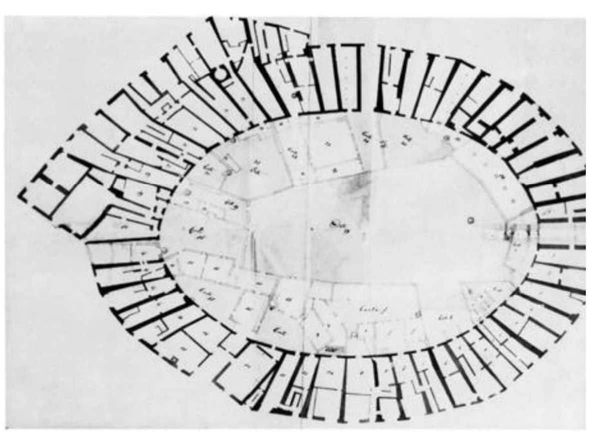
The interlocking and over layering of different fractal structures created at different periods. Note the Roman amphitheater shape

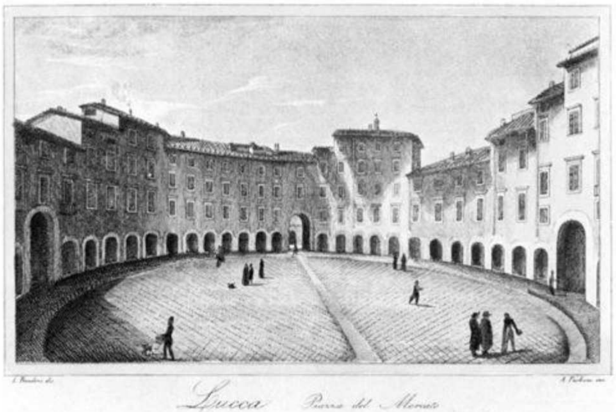
Source: Serge Salat, *Cities and Forms*

Arles and Lucca: Interlocking and over layering of geometries belonging to different morphological periods

Source: Serge Salat, *Cities and Forms*



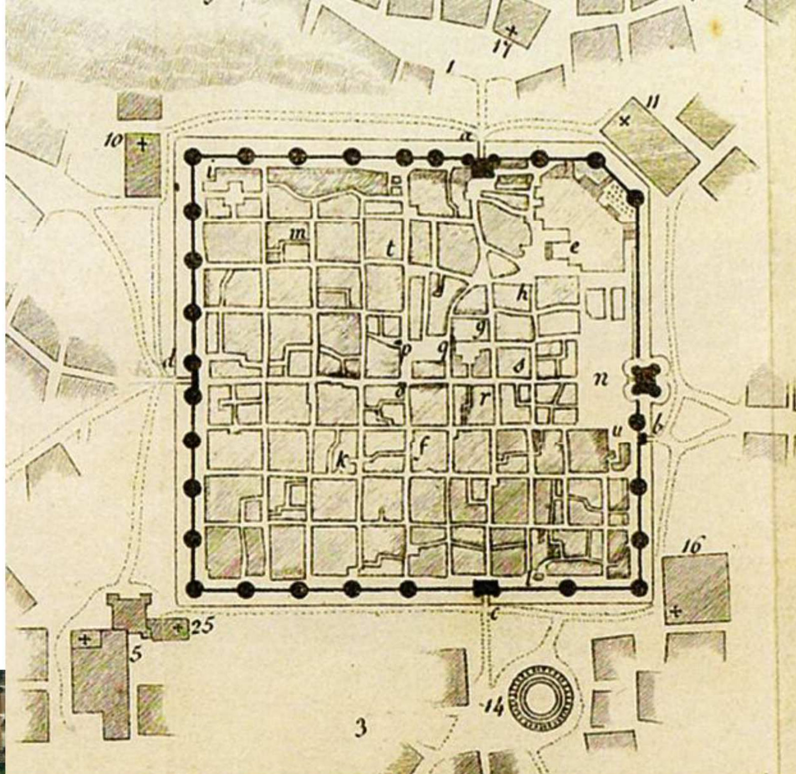


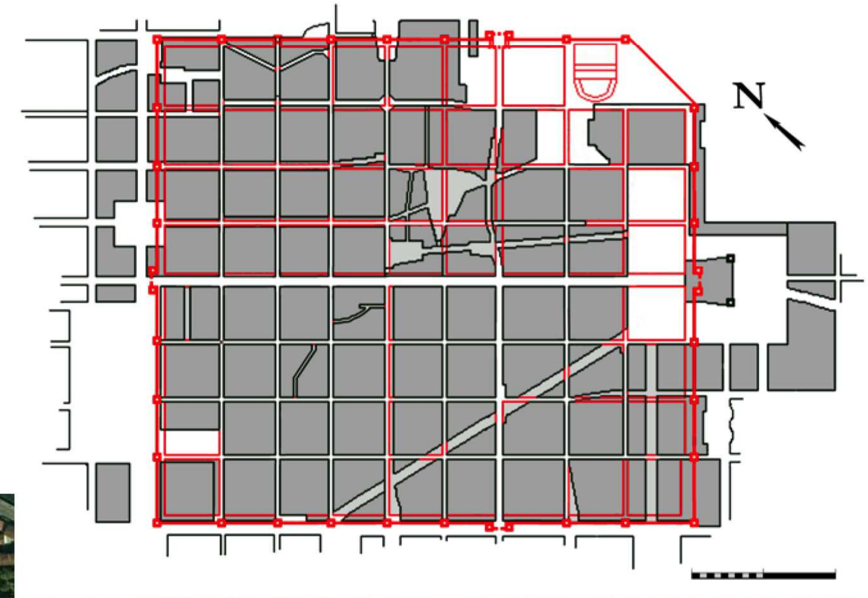


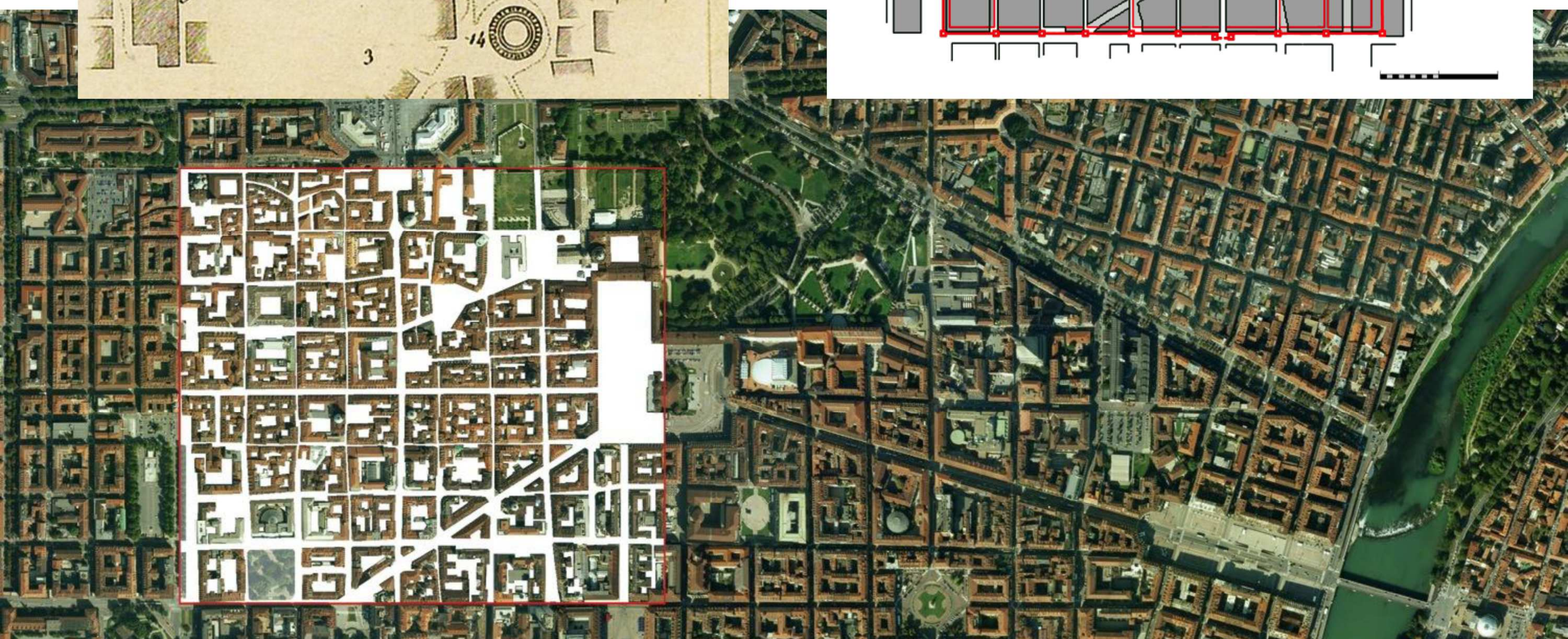
The resilience of street patterns

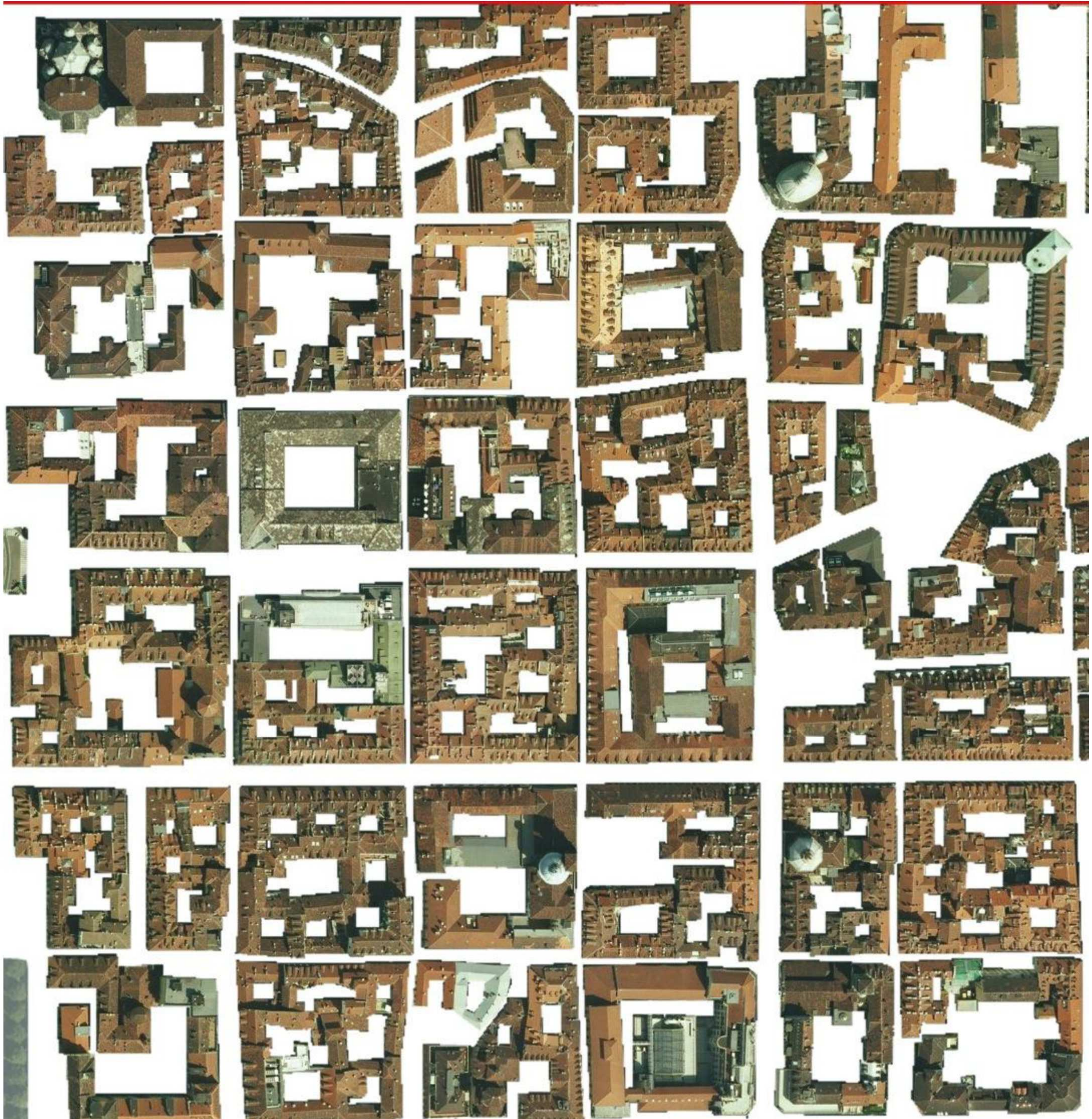
Torino in the roman period, in 1416 and today

Source: Serge Salat, *Cities and Forms*





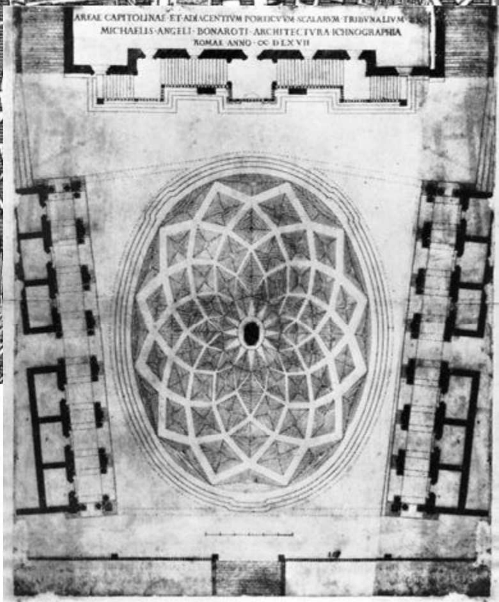




The diversification within the grid and the fractal increase of interfaces.
Torino in a 400 m side square
On 770 m x 710 m the neighborhood derived from the Roman town displays 40 km of street façades and 16 km of façades on courtyards.
This is the result of the fractal increase of interfaces within a limited surface

Source: Serge Salat, *Cities and Forms*

Roma Nolli-Piranesi Plan 1748

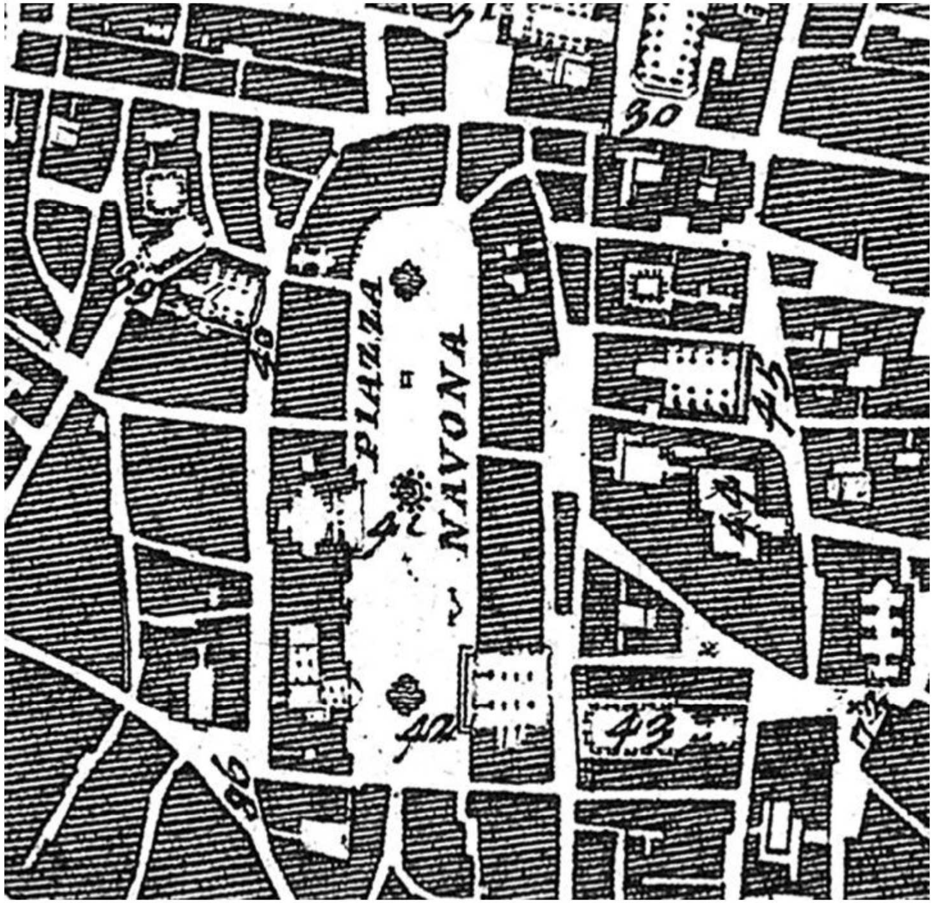


Source: Serge Salat, *Cities and Forms*

Around Piazza Navona today and in 1748: the imposition of a wider fractal urban layer on Roma 18th century urban fabric

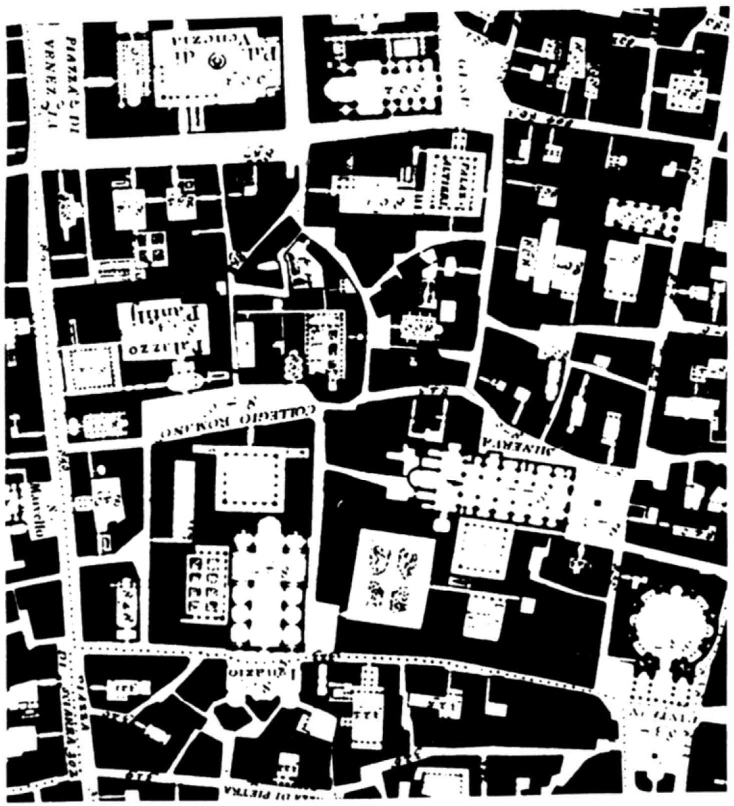
Source: Serge Salat, *Cities and Forms*

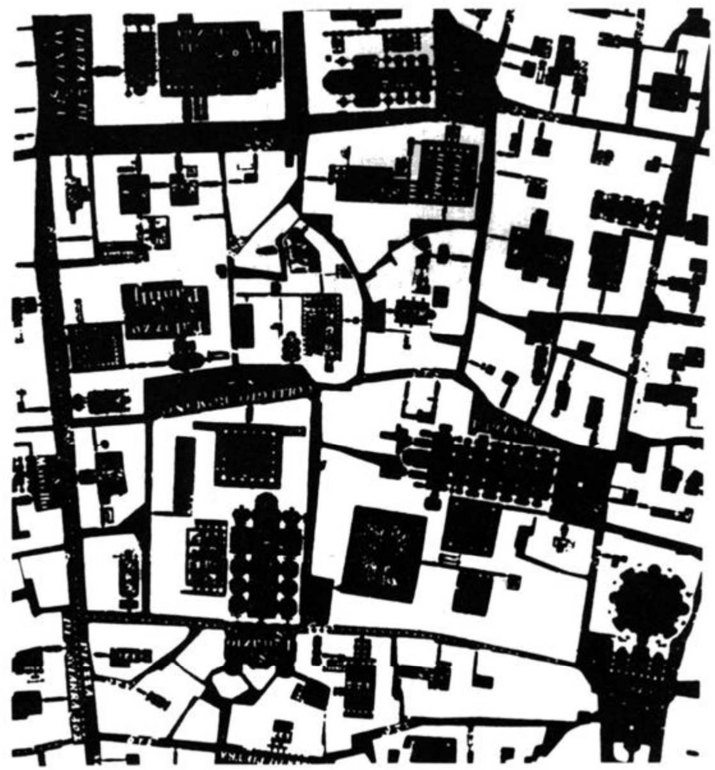
The subscale of plots, buildings and courtyards below the block scale



Source: Serge Salat, *Cities and Forms*

Positive and negative urban fractals: again an highly interlocked world

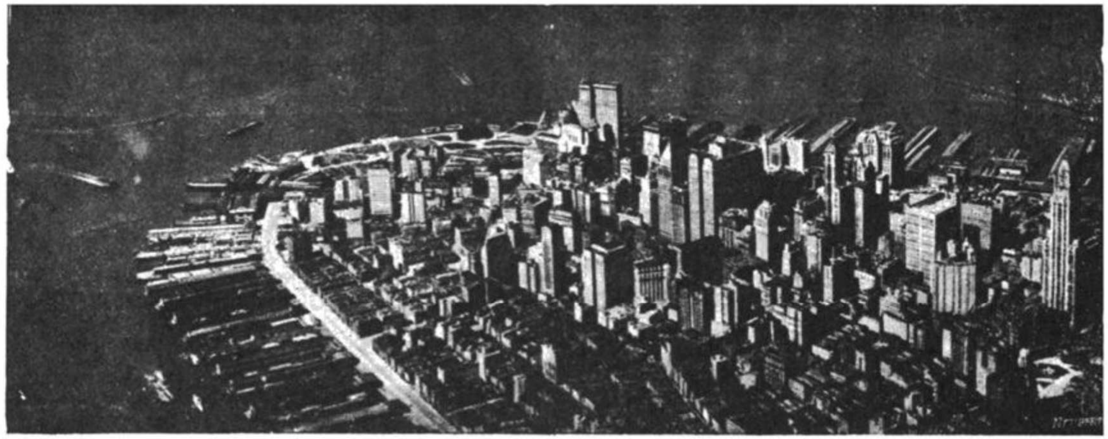


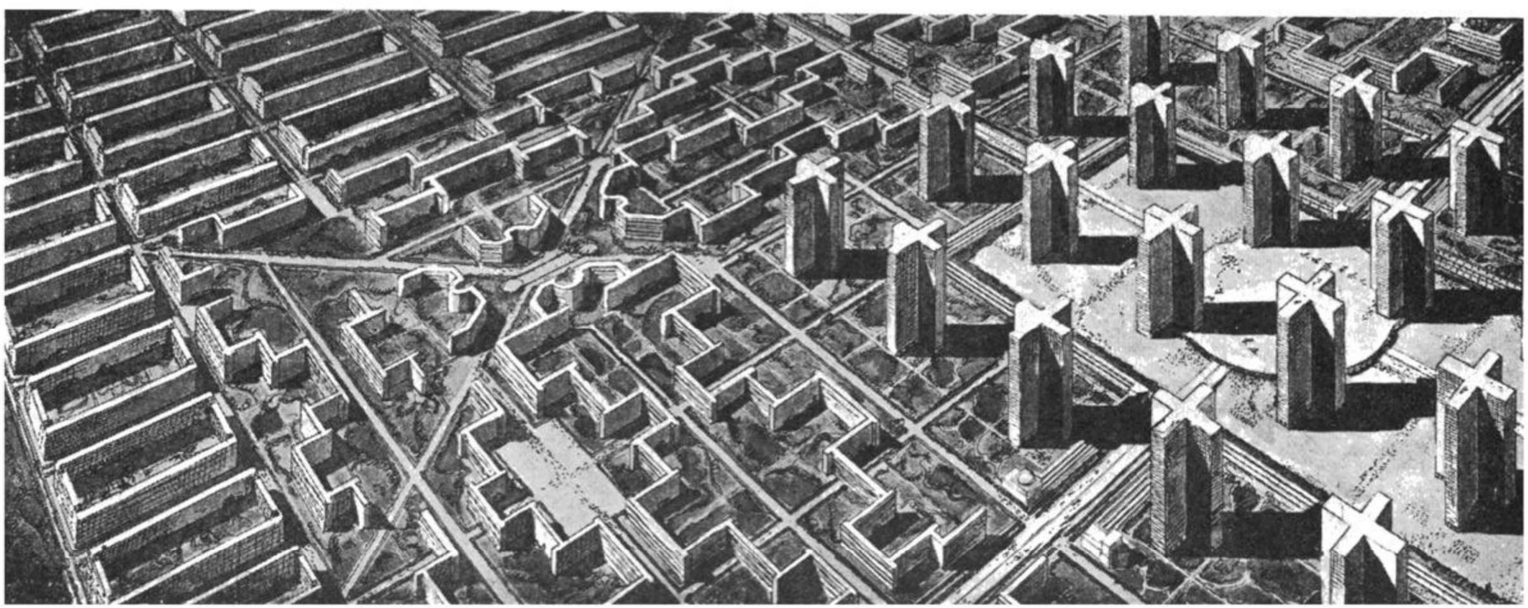


Source: Serge Salat, *Cities and Forms*

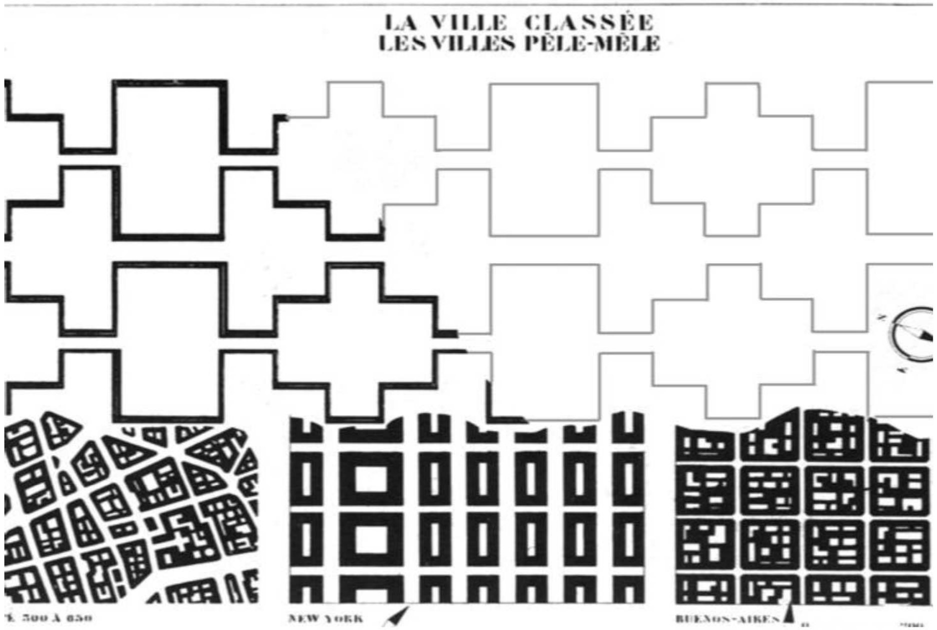
Modernism advocated the death of micro scales in urban multifractals

A la même échelle et sous un même angle, vue de la *Cité* de New-York et de la Cité de la « Ville contemporaine ». Le contraste est saisissant.



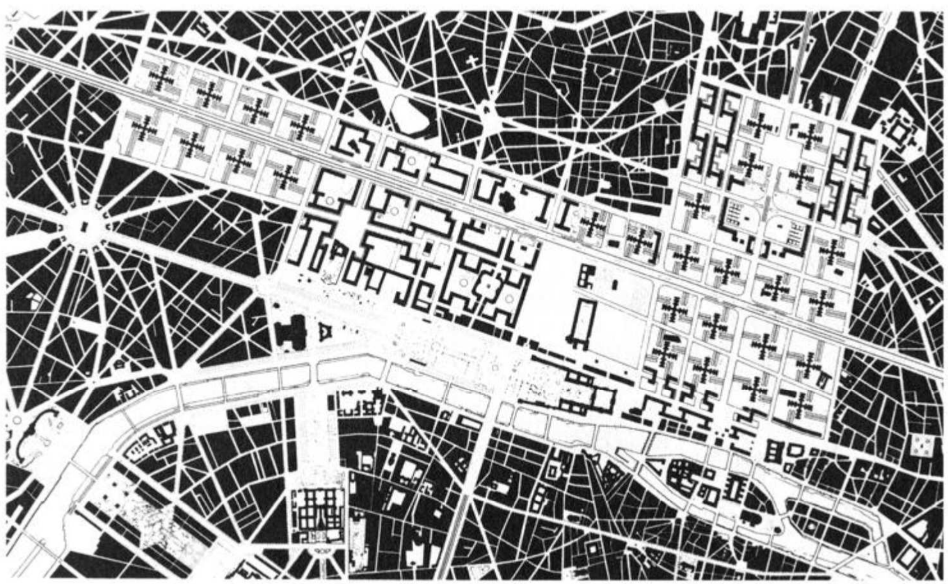


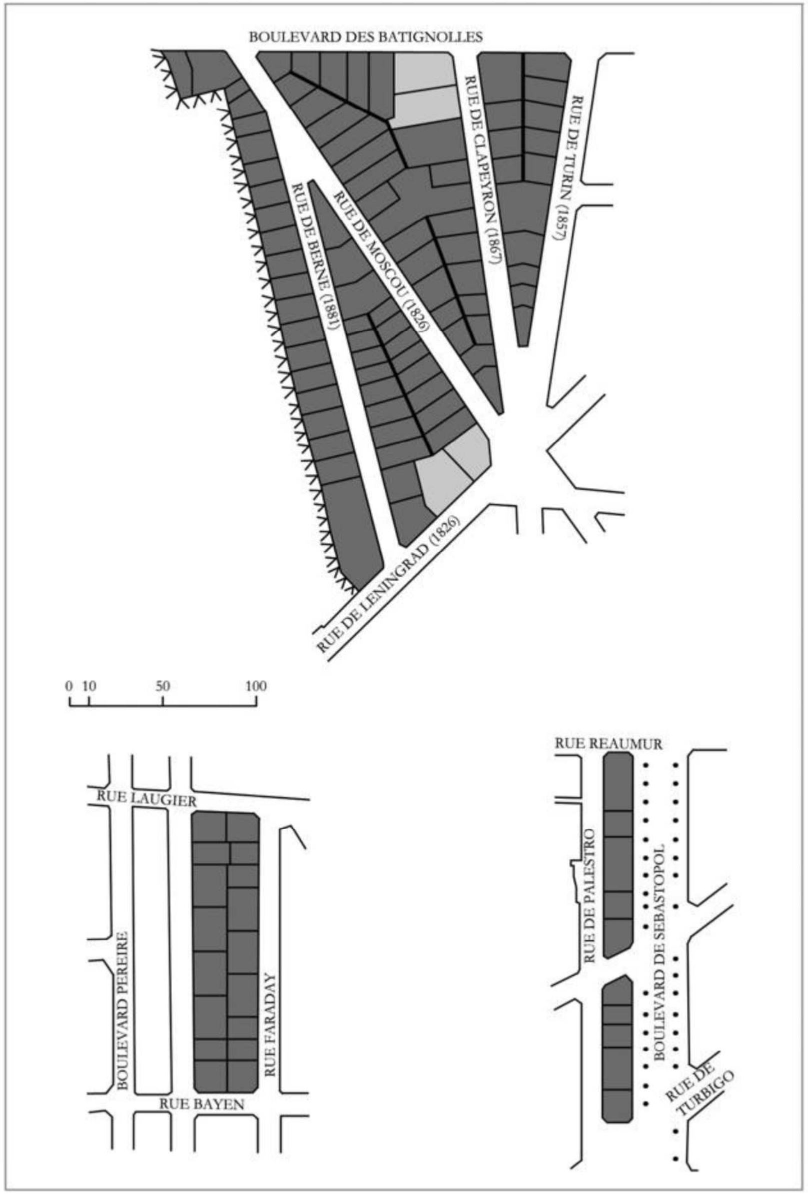
Le Corbusier did not like fractal cities. He called them “ La ville pêle-mêle “ and opposed them to the ordered “Radiant city” Source: Le Corbusier, *Urbanisme*

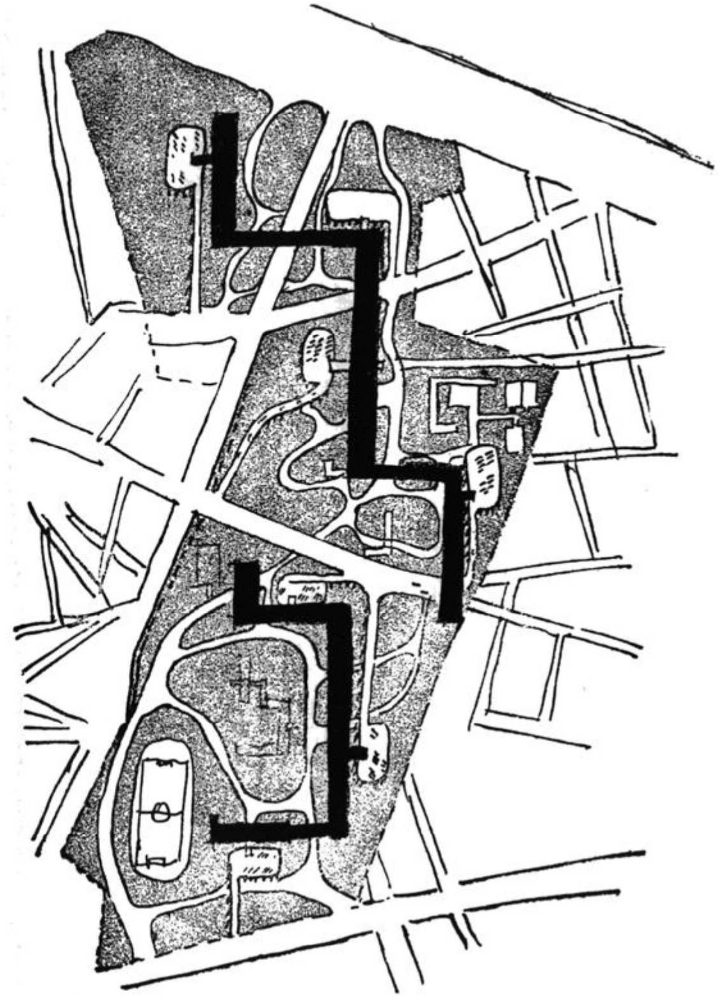


Modernism is the triumph of Euclidean order (*La ville classée*) over urban multifractals (*Les villes pêle-mêle*)

Source: Le Corbusier, *Urbanisme*





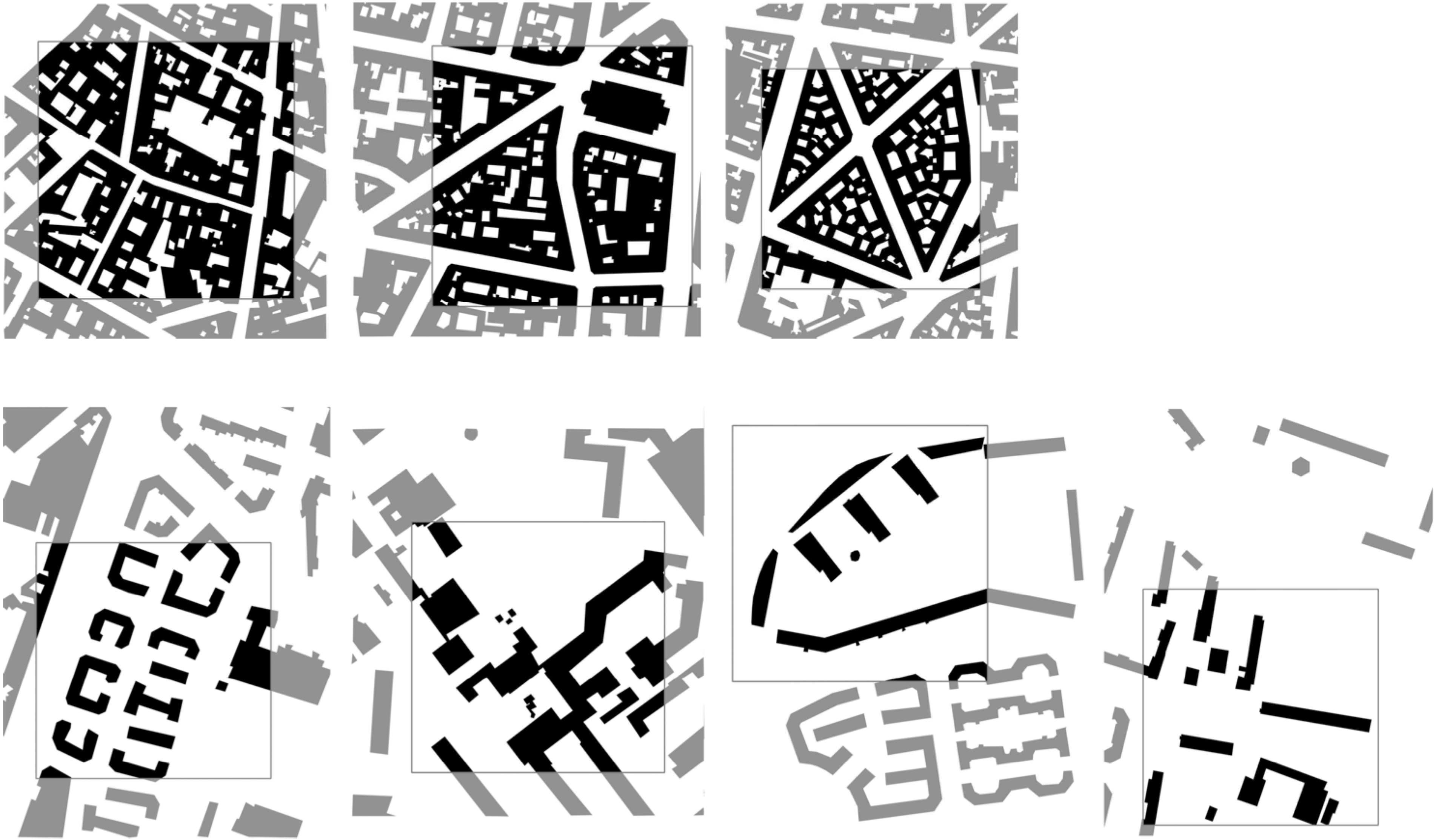


Le Corbusier’s vision for Paris

The agony of the urban block and of the urban multifractal struture

« Il faut tuer la rue » (Le Corbusier)

Source: Serge Salat, *Cities and Forms* and Le Corbusier *Urbanisme*



Le Corbusier’s modernist vision implemented in Paris intra-muros
Top: urban blocks from 18th century to 19th century (each central square: 200 m side)
Bottom: 20th century, from early century HBM to 1960’s near périphérique (each central square: 200 m side) Source: Serge Salat, *Cities and Forms*

Mathematical regularities emerge in almost all urban phenomena

Inverse power laws are the « signature » of complexity at all scales.

They derive from historical layering (Paris) or market forces (New York)

Multifractal urban spatial structures ensure <u>static optimization</u>

- energy efficiency
- economic efficiency
- social inclusion

Multiconnected urban forms ensure <u>dynamic optimization</u>

- better management of variable flows (traffic & energy)
- responsiveness to market fluctuations

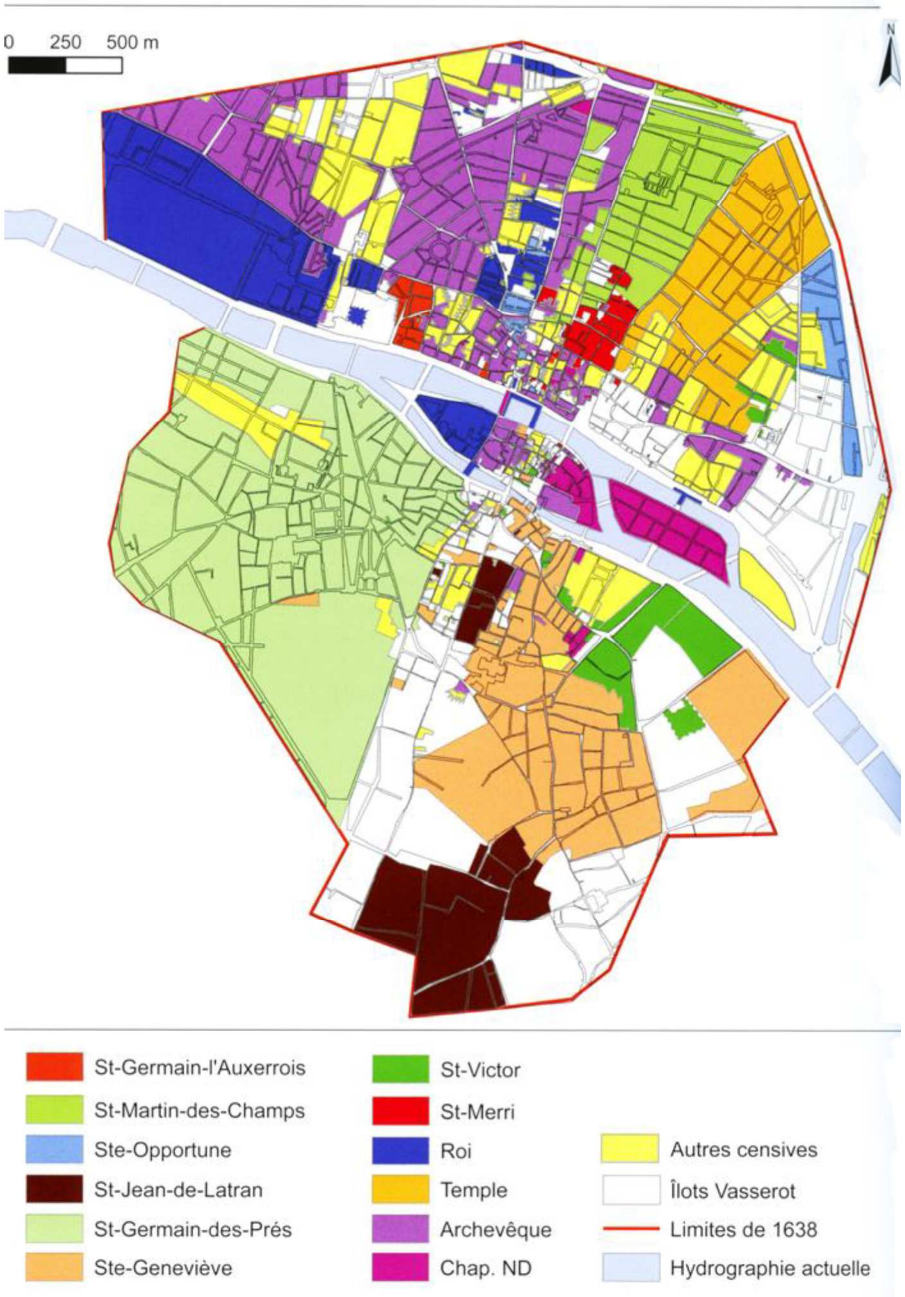
Paris

2000 years of urban evolution

have created a multifractal fine grain platting structure embedding the memory of all its history and a scale free street pattern

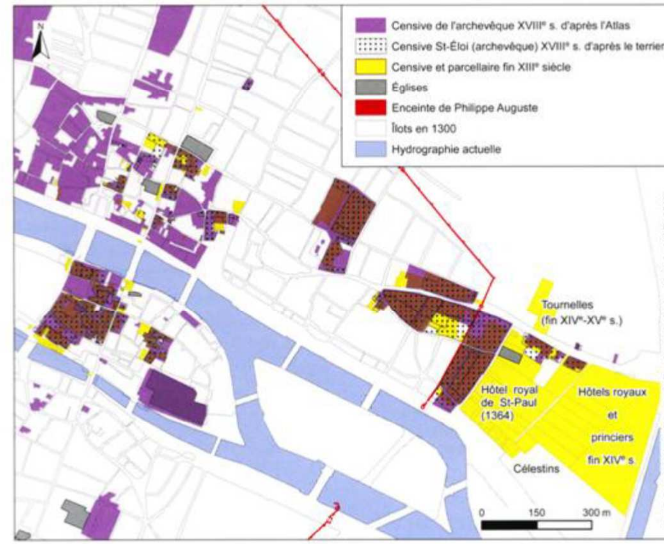
Paris on 1 square mile
a multifractal structure shaped by 2000 years of urban evolution

Source: Serge Salat, *Cities and Forms*

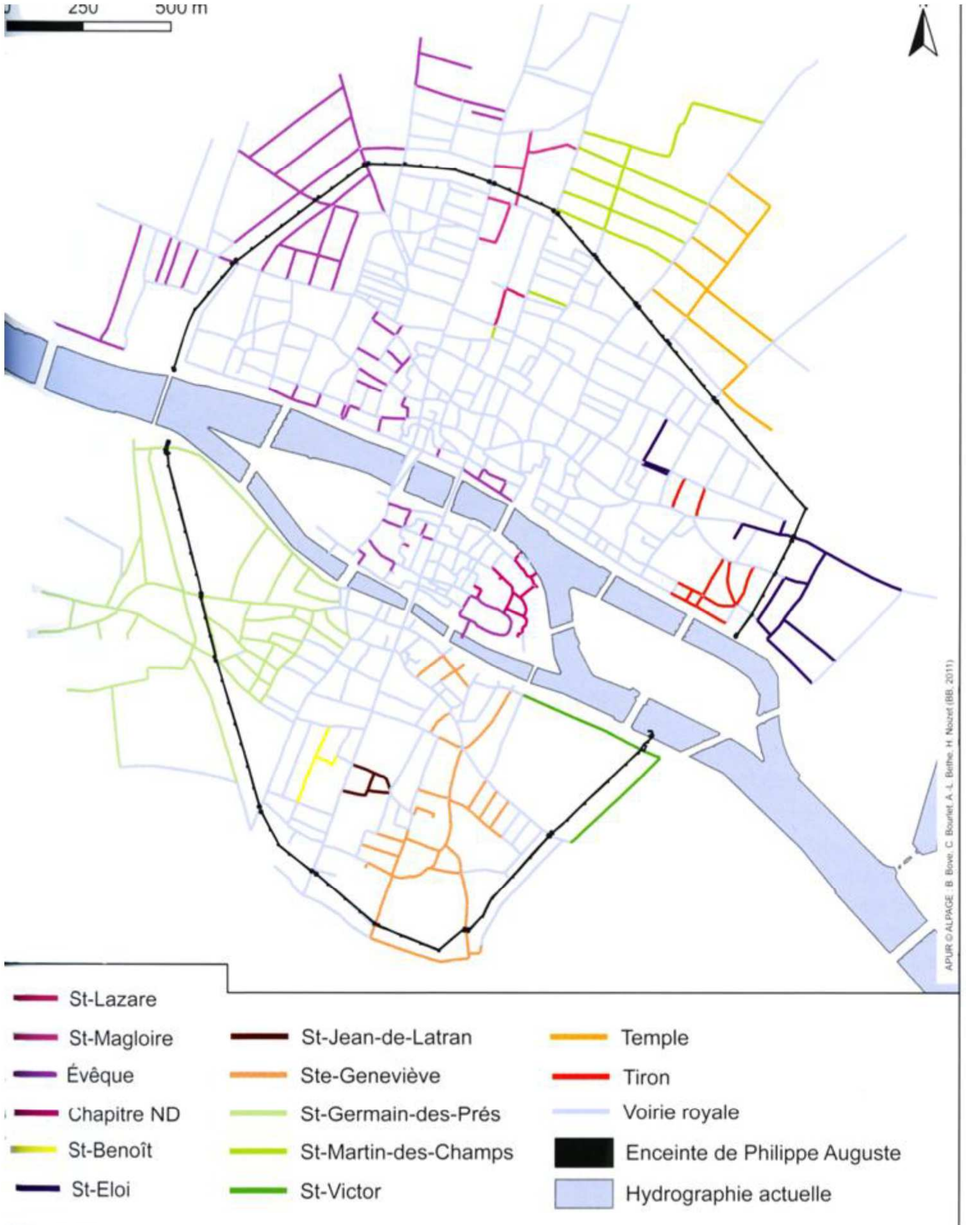


Source: Hélène Noiset, Boris Bove, Laurent Costa (dir), *Paris de parcelles en pixels*

Urban powers fragmentation: "Censives" in Paris XVIIth century



Paris land was, from the early Middle Ages until the revolution, divided between many lordships. The lords gradually granted to individuals settled on their land tenures on which they perceived an annual fee, the "cens" - hence the name "censive" for Paris lordships. This property tax recognized the eminent property of the Lord on the land, the tenant having to settle for the useful property of the plot. This eminent property gave a number of rights to Lords: land rights as the perception of the "cens" or transfer duties, but sometimes political rights such as rights on roads or high, middle and low justice. The Lordship fact was therefore an essential element of the urban life of the Ancient Regime. It gradually became a framework within which social life took place.



Religious and feudal power on streets in 1300 Paris

Compared to the previous map showing the large rural religious lordships and the smaller and more fragmented urban religious lordships, with little control of the king on Paris land, this map shows that the king had acquired in 1300 the lordship of a great part of the streets.

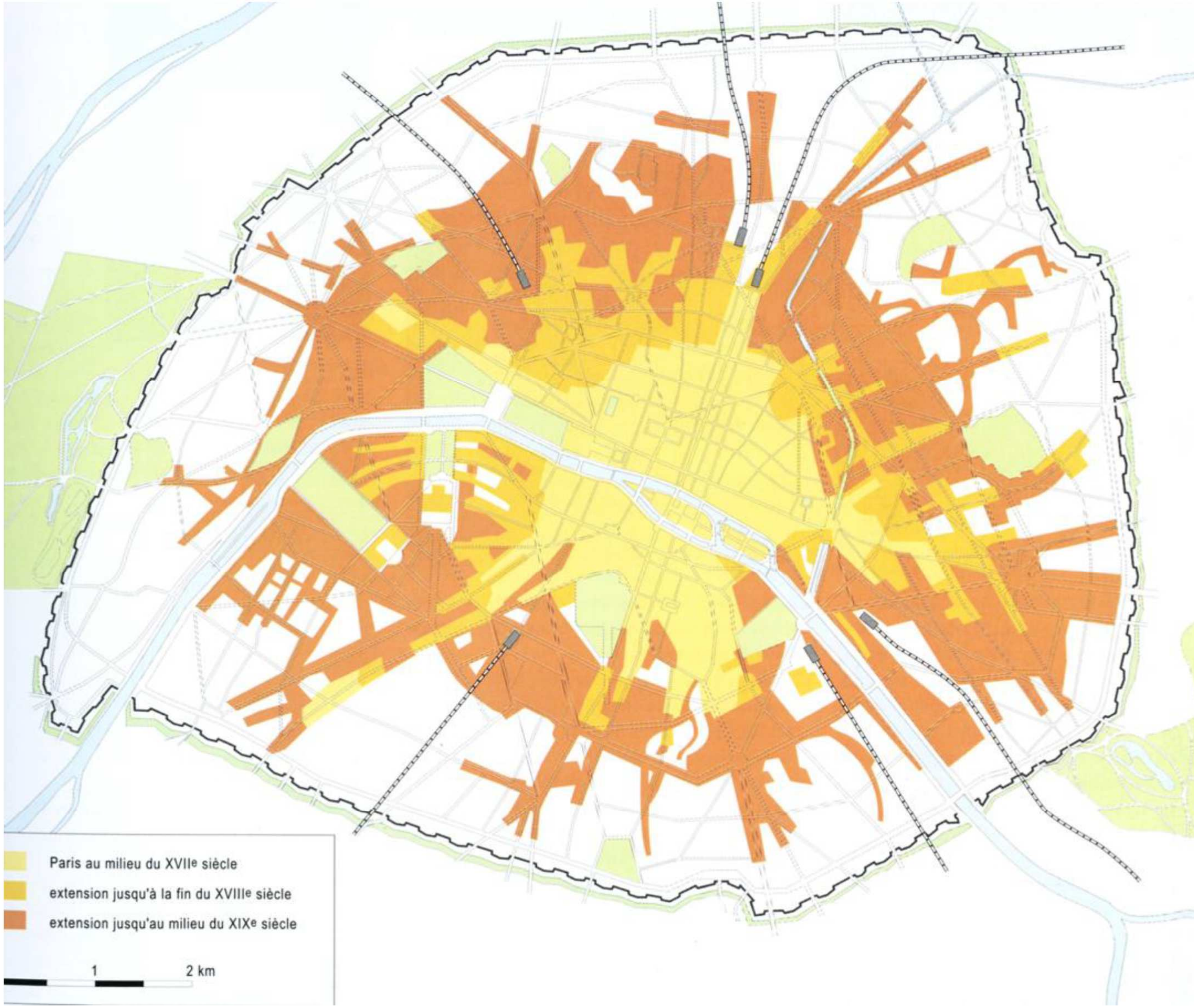
Source: Hélène Noiset, Boris Bove, Laurent Costa (dir), *Paris de parcelles en pixels*

The successive medieval walls of Paris:

- The wall of 10th century
- The wall of Philippe Auguste (1190-1215)
- The wall of Charles V (1356)

The successive walls have had a morphological impact on the fine structure of the city.

Source: Hélène Noiset, Boris Bove, Laurent Costa (dir), *Paris de parcelles en pixels*



Paris extension until the 19th century is not strictly radio-concentric. Since the end of the Middle Ages, the main suburbs (Saint-Honoré, Saint-Denis and Saint-Martin, Saint-Antoine) grow linearly. Source: Hélène Noiset, Boris Bove, Laurent Costa (dir), *Paris de parcelles en pixels*

Paris plot size pattern is a legacy of the medieval period, not Antiquity: while the Roman period city is clearly focused on the left bank, the main urban center has developed on the right bank from the Middle Ages in a tripartition : Université - Cité - Ville.

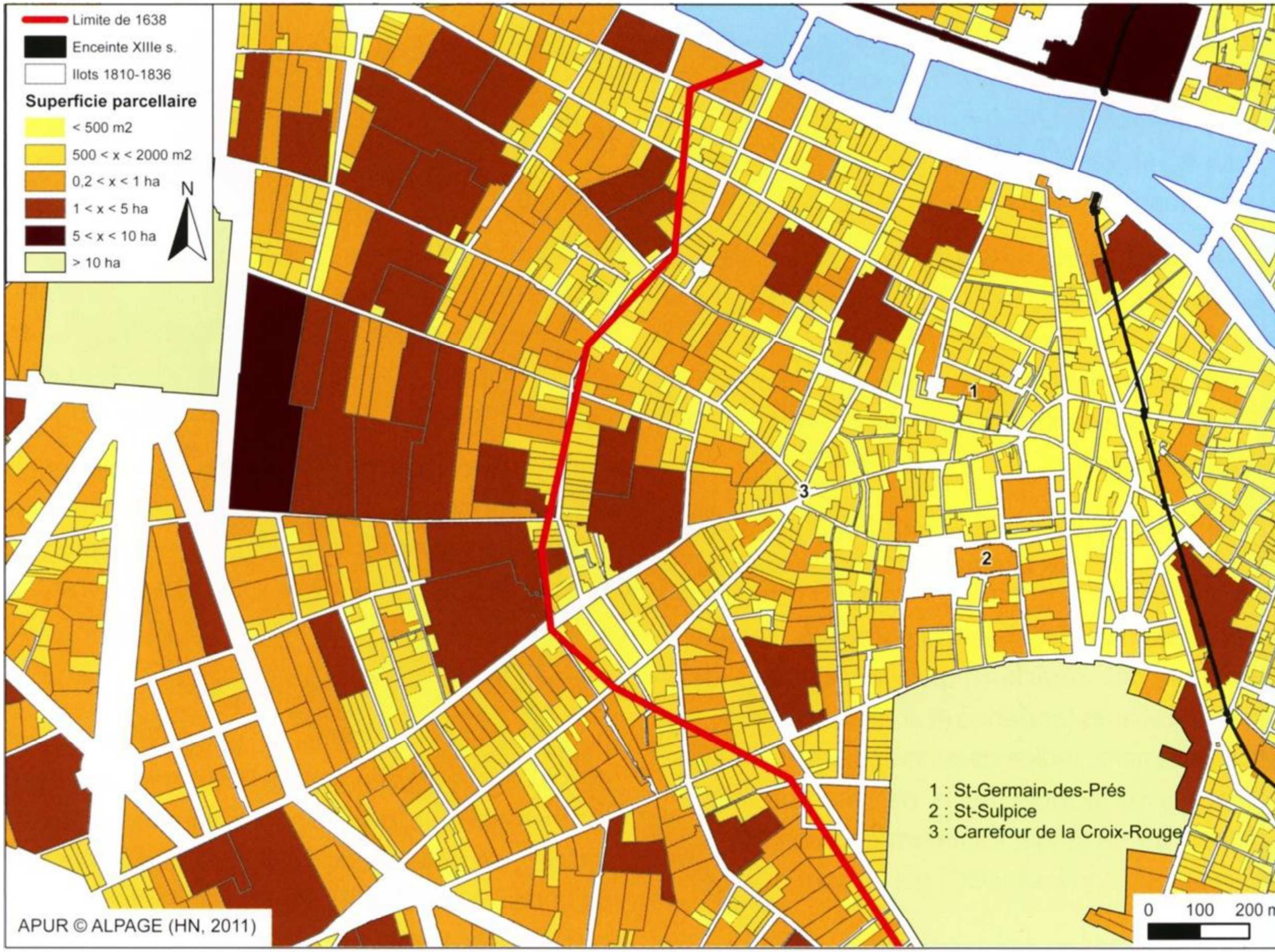
The polygonal characterization of plots involves the exploitation of the geometric characteristics of past and present plot layers:

- surface ,
- elongation index (ratio length/width) ,
- index of rectangularity reflecting the surface of the plot considered in relation to the minimum bounding rectangular box and the convex envelope associated with it.

Overall, the pre-industrial plot characteristic of medieval and modern periods, is in a range between 12 m^2 and 300 m^2 , with plots most often between 50 and 100 m^2 . Highlighting below 300 m^2 plots on the Vasserot plan (1810-1836) confirms the high plot density on the more urbanized right bank compared to the more rural left bank. Per hectare there was on average 11 plots on the right bank against 8 on the left bank.

	Aire totale (ha)	Nb de parcelles	Densité parcellaire (nb parc. /ha)	Aire parcellaire moyenne	Écart-type (aire moyenne)
ive droite	1676	18364	**11**	912,52	**3881**
ve gauche	940	7252	**8**	1296,62	**9118**

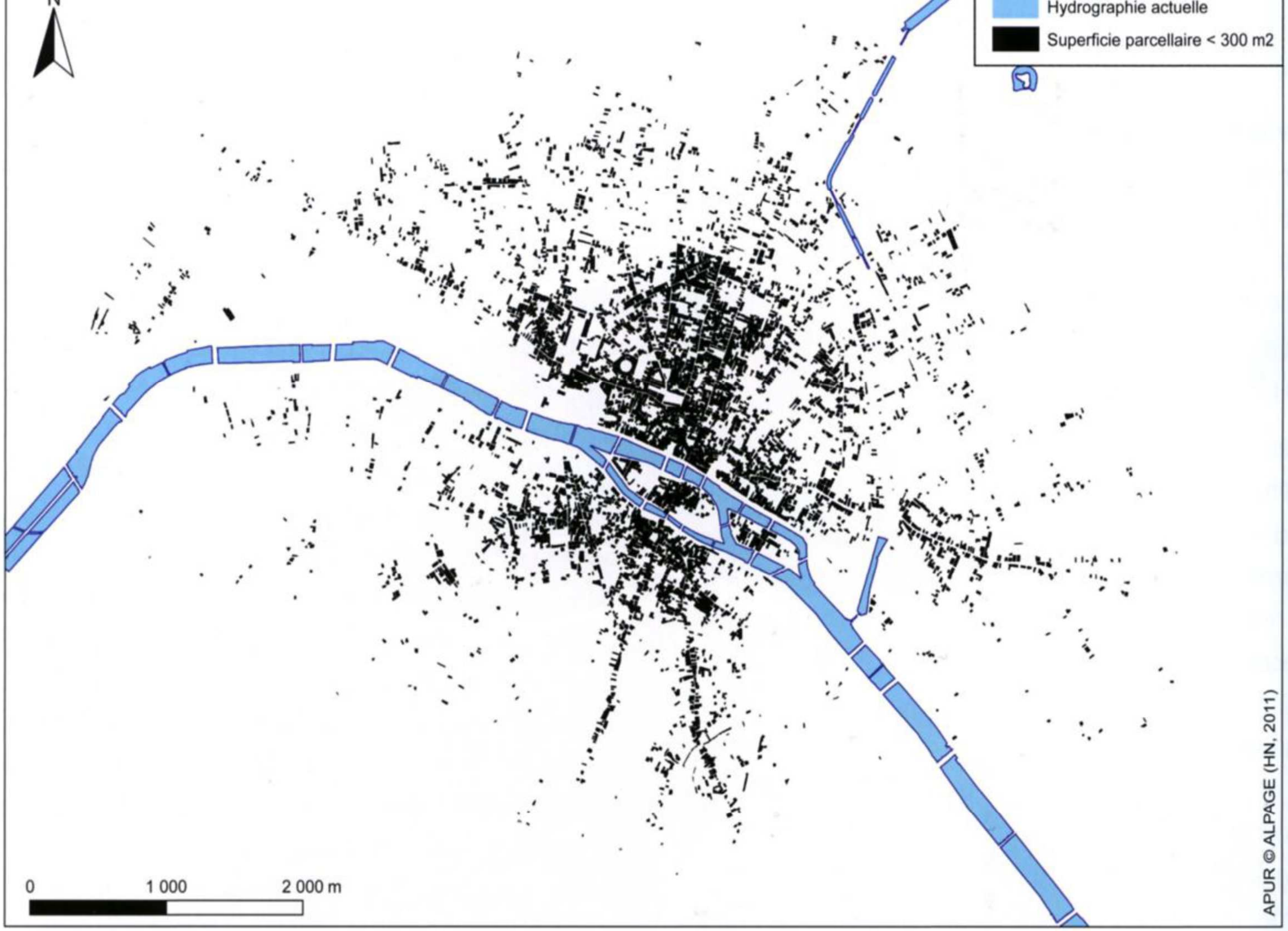
Source: *Paris de parcelles en pixels*



Articulation entre parcellaires rural et urbain vers Sèvres-Babylone.

The limit between urban and rural space on the left bank in the years 1810 - 1836 appears in the morphological difference in the plot patterns.

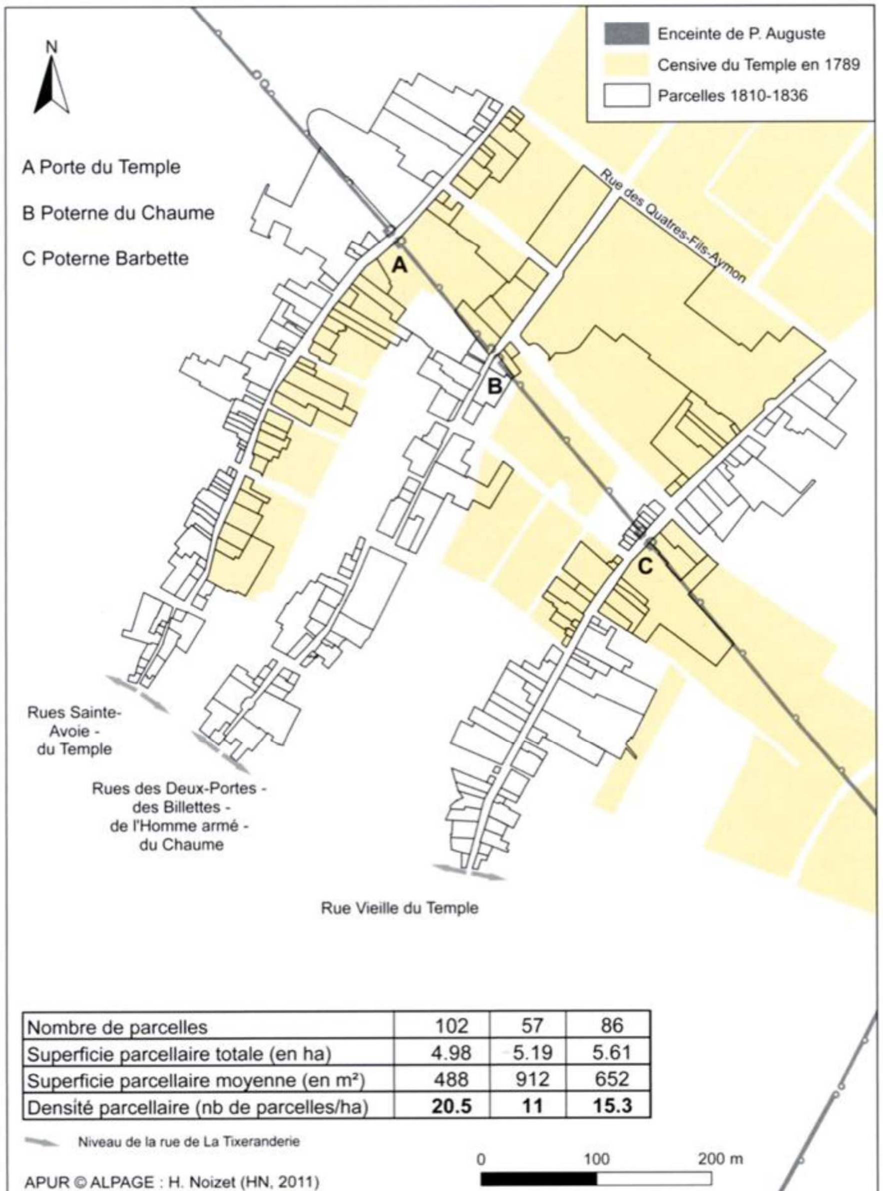
Source: *Paris de parcelles en pixels*



Extracting only the smaller plots corresponding to the most ancient urbanization reveals a fractal pattern oriented according to the 2 morphogenetic axes of Antiquity and the Middle Ages

Source of the map: *Paris de parcelles en pixels*

Platting geometry (size, orientation) is a time travel machine in layered urban strata
It embeds the memory of the city at extreme micro scales



Nombre de parcelles	102	57	86
Superficie parcellaire totale (en ha)	4.98	5.19	5.61
Superficie parcellaire moyenne (en m²)	488	912	652
Densité parcellaire (nb de parcelles/ha)	**20.5**	**11**	**15.3**

Consequences on the platting geometry of the opening dates of gates in Philippe Auguste wall (1190-1215)

- Rue du Temple crossed the wall through porte du Temple, one of the original gates
- -Rue Vieille-du-Temple was opened very early, before 1203
- - Rue du Chaume was opened only in 1288

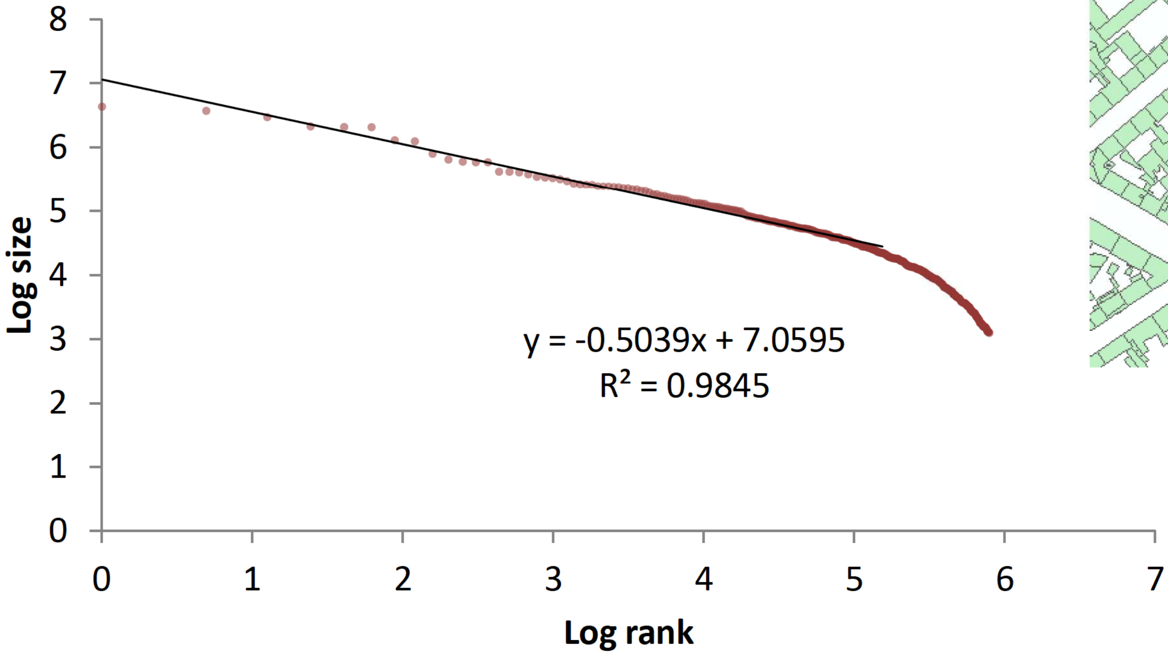
The analysis of plots in Vasserot plan (1810-1836) reveals a morphological hierarchy with 20.3 plots per ha for rue du Temple, 15.5 plots per ha for rue Vieille-du-Temple, 11.3 plots per ha for rue du Chaume.

The piercing of wall gates has been so structuring on the micro scale of the urban structure that 5 centuries after, at the beginning of 19th century, the spatial hierarchy of 13th century is still visible.

Source: *Paris de parcelles en pixels*

The scaling hierarchy of plots' area along a structuring axis of Medieval Paris
The largest plot is 760 m²

Paris
Rue Mouffetard



Source: Urban Morphology Institute

Quartier de l'Etoile

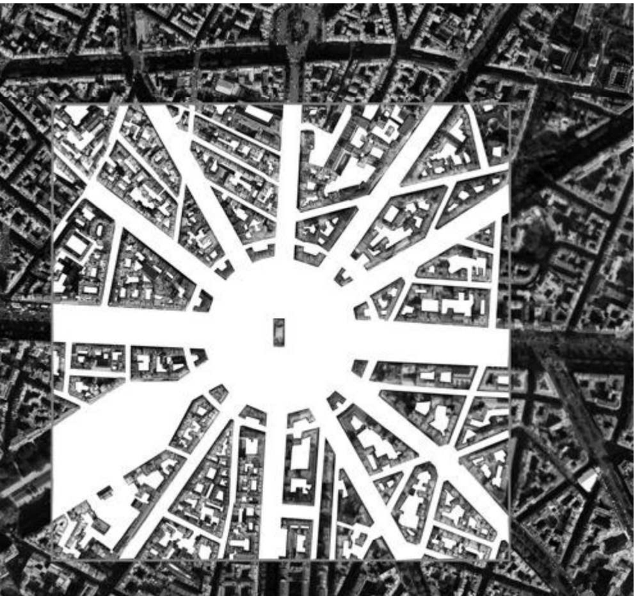
The plot scaling hierarchy in a new Haussmannian development.

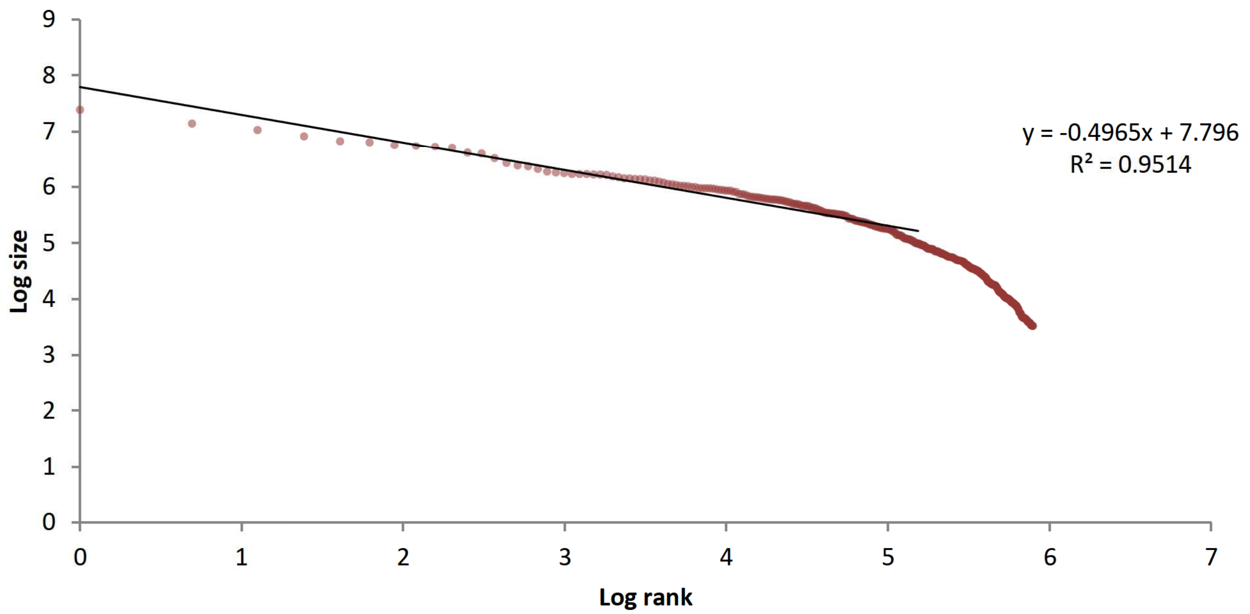
The largest plot is 1600 m^{2}.

With much larger plots the scaling hierarchy remains similar.

The city dilatation conserves the scaling hierarchy

Source: Urban Morphology Institute

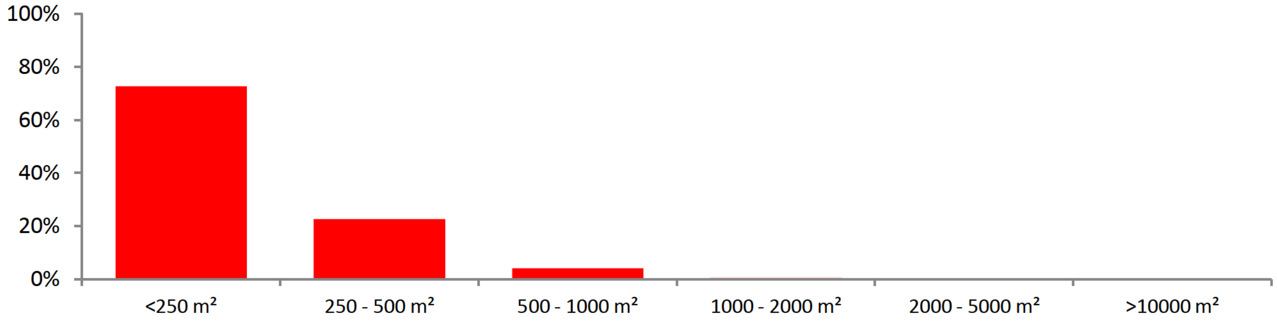




The plot scaling hierarchy in a complex urban fabric near Nation.

Source: Urban Morphology Institute

Paris
Quartier sainte Marguerite

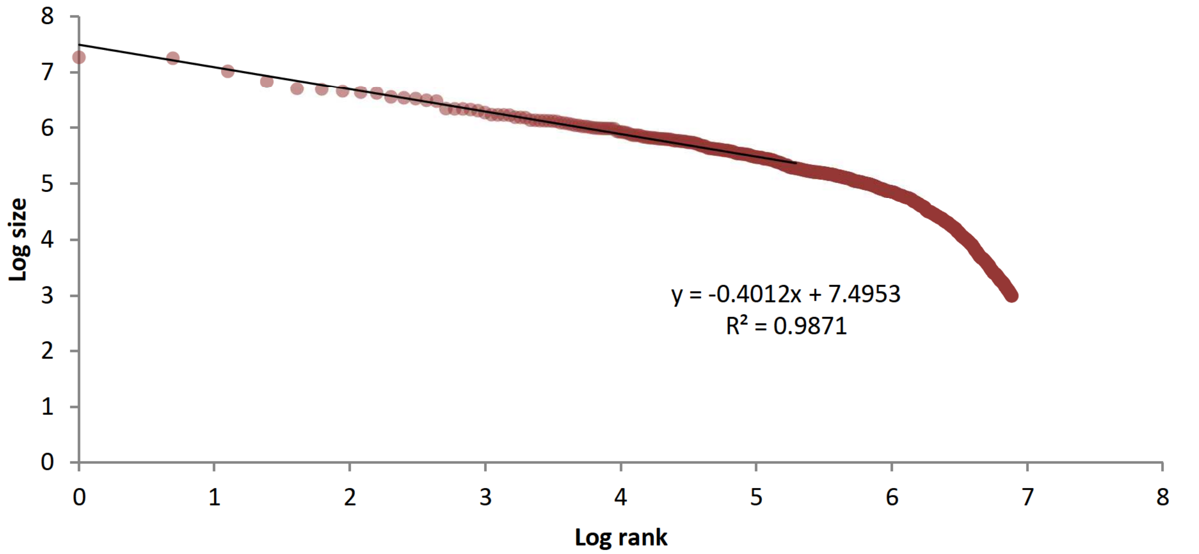


With much larger plots (3300 m2 for the largest) the scaling hierarchy remains similar.

Is this plot scaling exponent a signature of Paris intra-muros?

Source: Urban Morphology Institute

Paris
Quartier sainte Marguerite

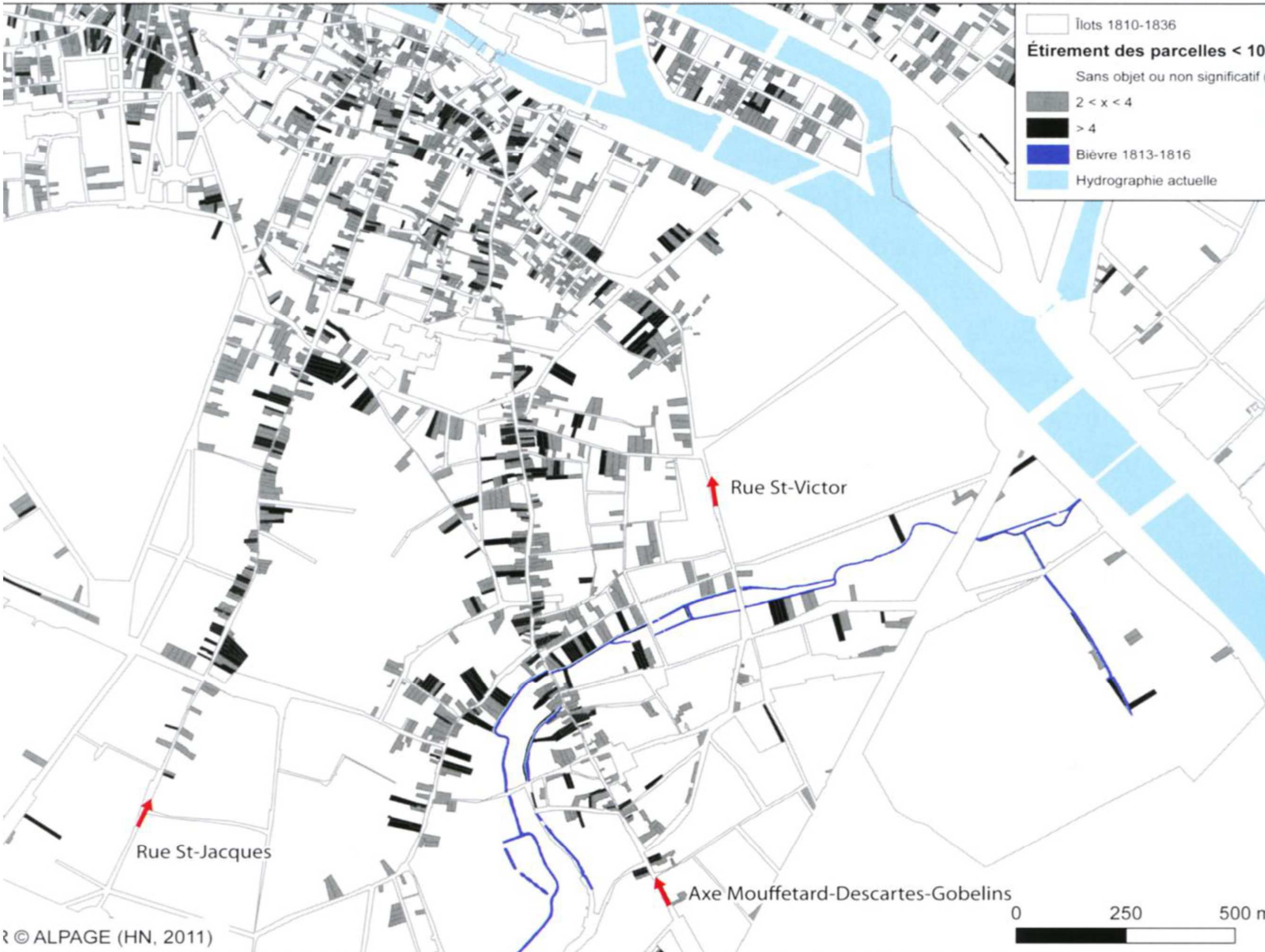


The analysis of the plot elongation index
The elongation index allows to measure the attractiveness of streets.

A strong elongation characterizes a plot on a street along high traffic routes .

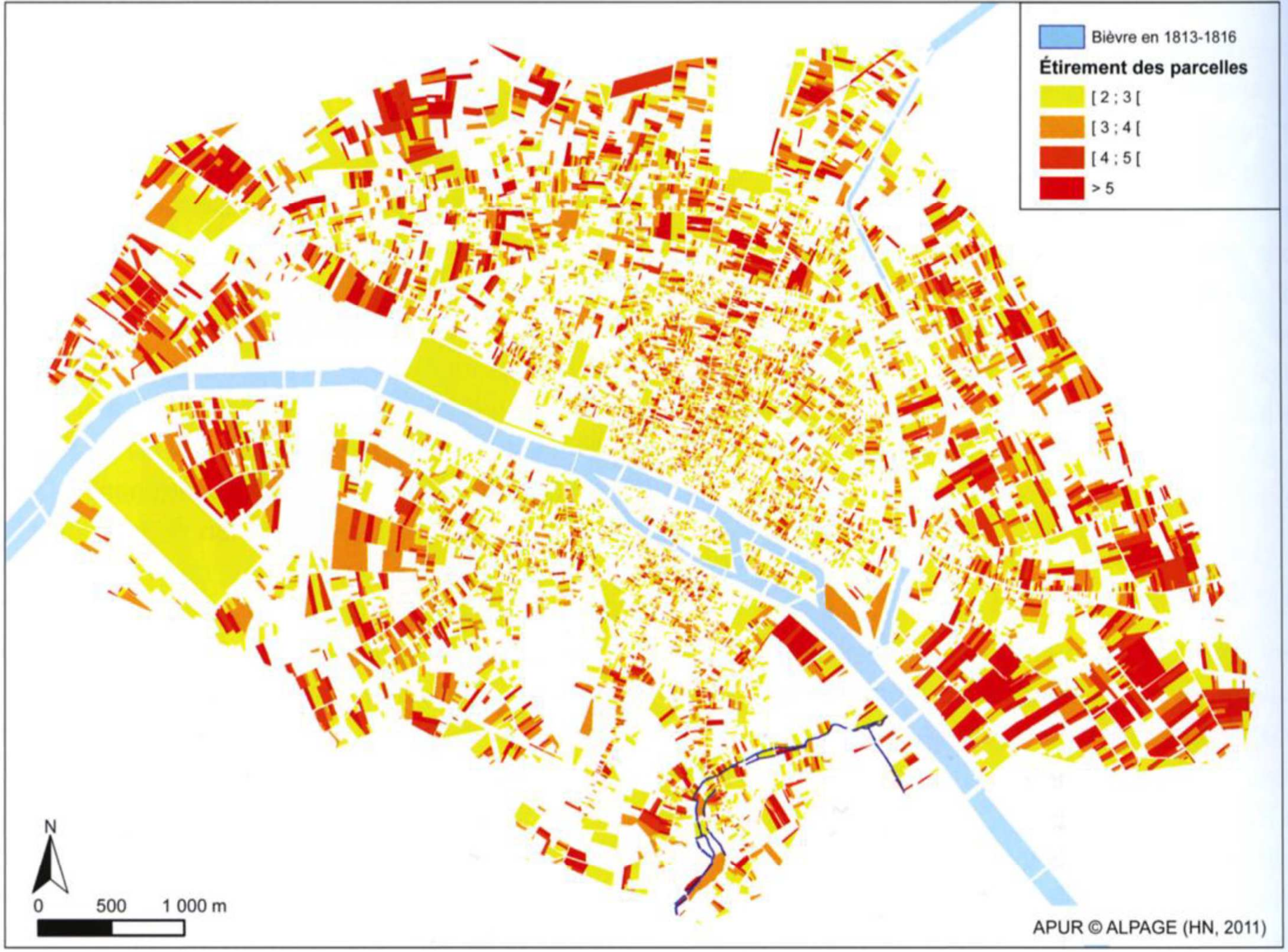
The ability of some streets to capture flows crystallizes in the plot configuration into strips or fringes. The added value of the land bordering these streets, which reflects the competition in order to have access to them, induces a dense, patchy and fragmented plot pattern with short street facade and deployment behind the plot.

This strip plot morphology characterizes rue Saint- Jacques – the former axis linking the North and South – and rue Mouffetard , the former axis that was on the route Paris - Melun. South of the wall of Philippe Auguste , these two axes induced a suburban type configuration with very loose plots articulated around a strong axis where plots were, in contrast, very fragmented.



Étirement des parcelles 1810-1836 en rive gauche :
rôle structurant des rues Saint-Jacques et Mouffetard et de la Bièvre.

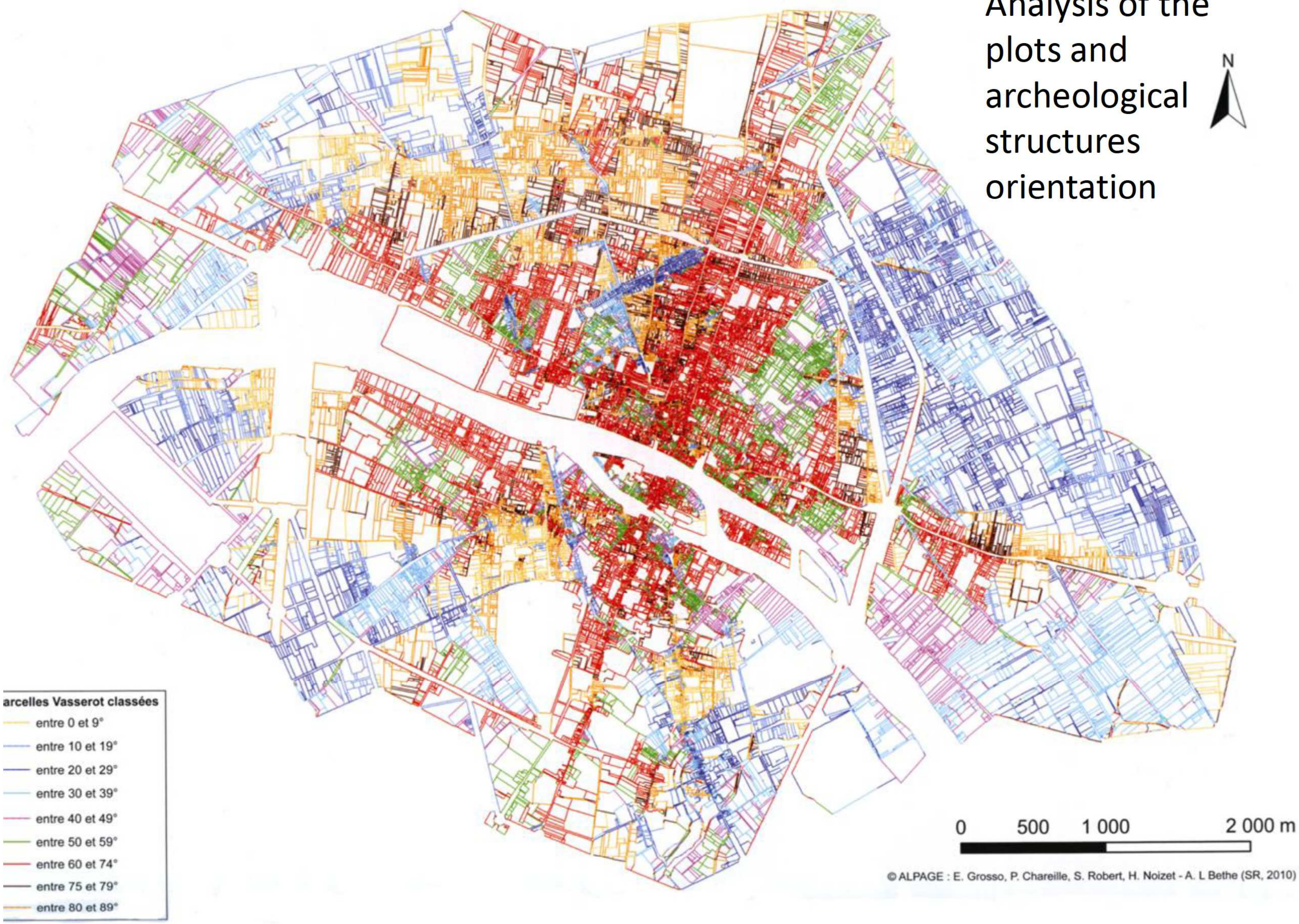
Source: *Paris de parcelles en pixels*



Étirement des parcelles 1810-1836 : un parcellaire rural en bandes en périphérie.

The elongation index also identifies strip plots of rural origin. This is the type of plot that is located on the outskirts of Paris from before 1860, with a regular arrangement of strip flooring, which corresponds to intermediate forms of agricultural land use. In addition, the elongation index also identifies the mechanisms of plot transmission for earlier periods, such as between rue Saint-Martin and rue Vieille-du-Temple. Source: *Paris de parcelles en pixels*

Analysis of the plots and archeological structures orientation

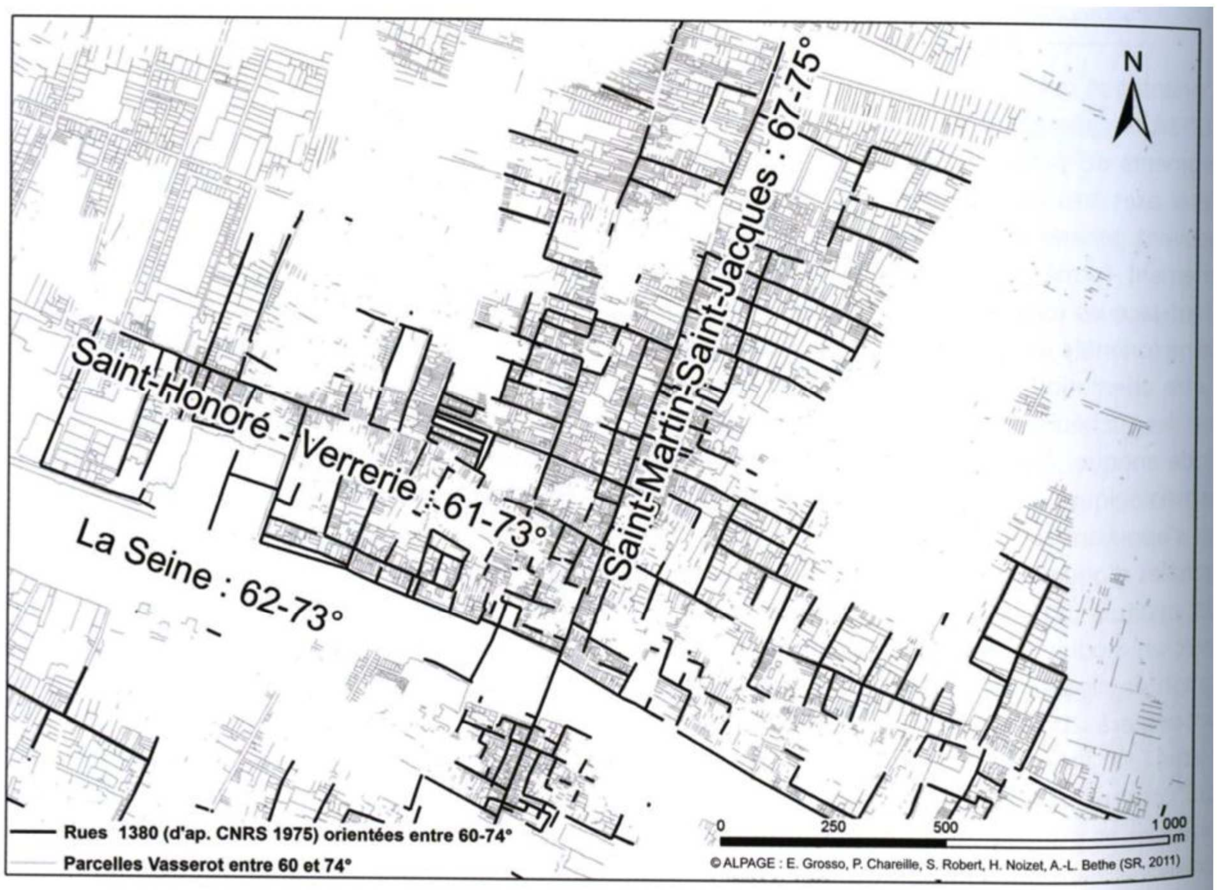


This map shows the orientations of the segments of plots in Vasserot map (1810-1836) as well as archaeological structures of Paris .

Source: *Paris de parcelles en pixels*

The major orientation is between 60 and 74 ° with respect to east. It alone represents 36 % of the total of segments. It relies on two very morphogenetic axes , that is to say, that can generate and transmit forms: the alignment formed by the rue Saint -Martin and Saint- Jacques, and the Seine. **This orientation has been identified by archaeologists as dominant in the Roman period .**

The morphogenetic axis of ancient Lutèce was based on a regular orthogonal grid aligned on rue Saint- Martin – rue Saint- Jacques, which is partly the cardo of the ancient foundation and builds on former islands formerly present in the course of the Seine.



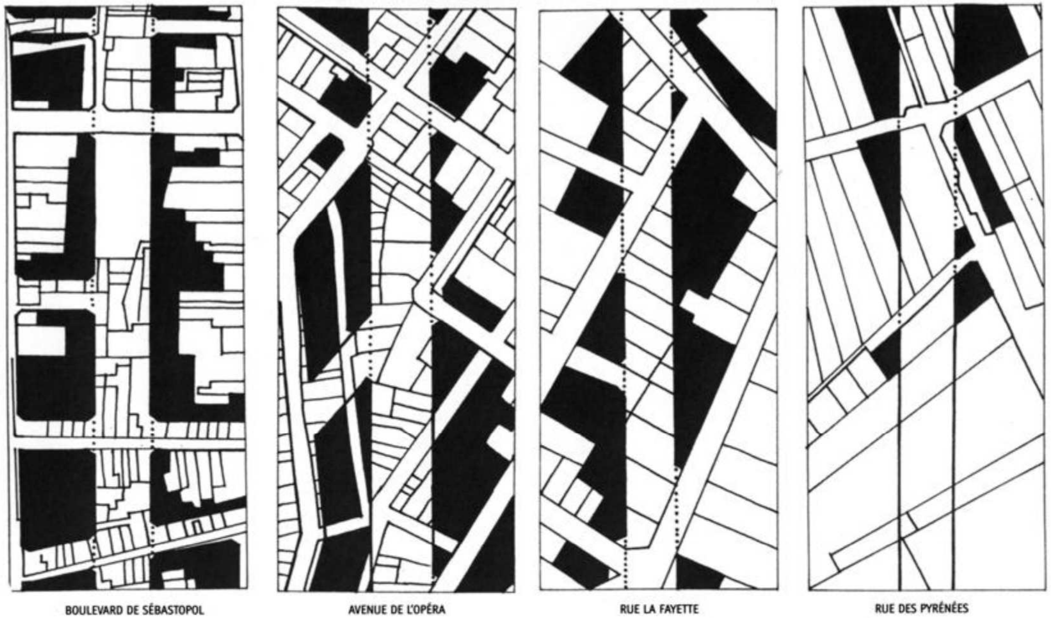
This orientation also dominates the network of streets that existed at the end of the fourteenth century. T**he Middle Ages has played a key role in the resilience of Roman period main orientation and its dissemination on the right bank**.

Source: *Paris de parcelles en pixels*

Percées haussmanniennes et voies anciennes : une organisation différente du parcellaire actuel en front de rue

The rectangular shape is the basic plot module. Thus, the indices measuring greater or lesser compliance with this standard are instructive on the structuring of the urban fabric. Deviations from the standard rectangular plot highlight planning actions that operate cuts in the continuity of the urban fabric, as here Haussmann cuts that have produced a large number of indented plots.

Source: *Paris de parcelles en pixels*



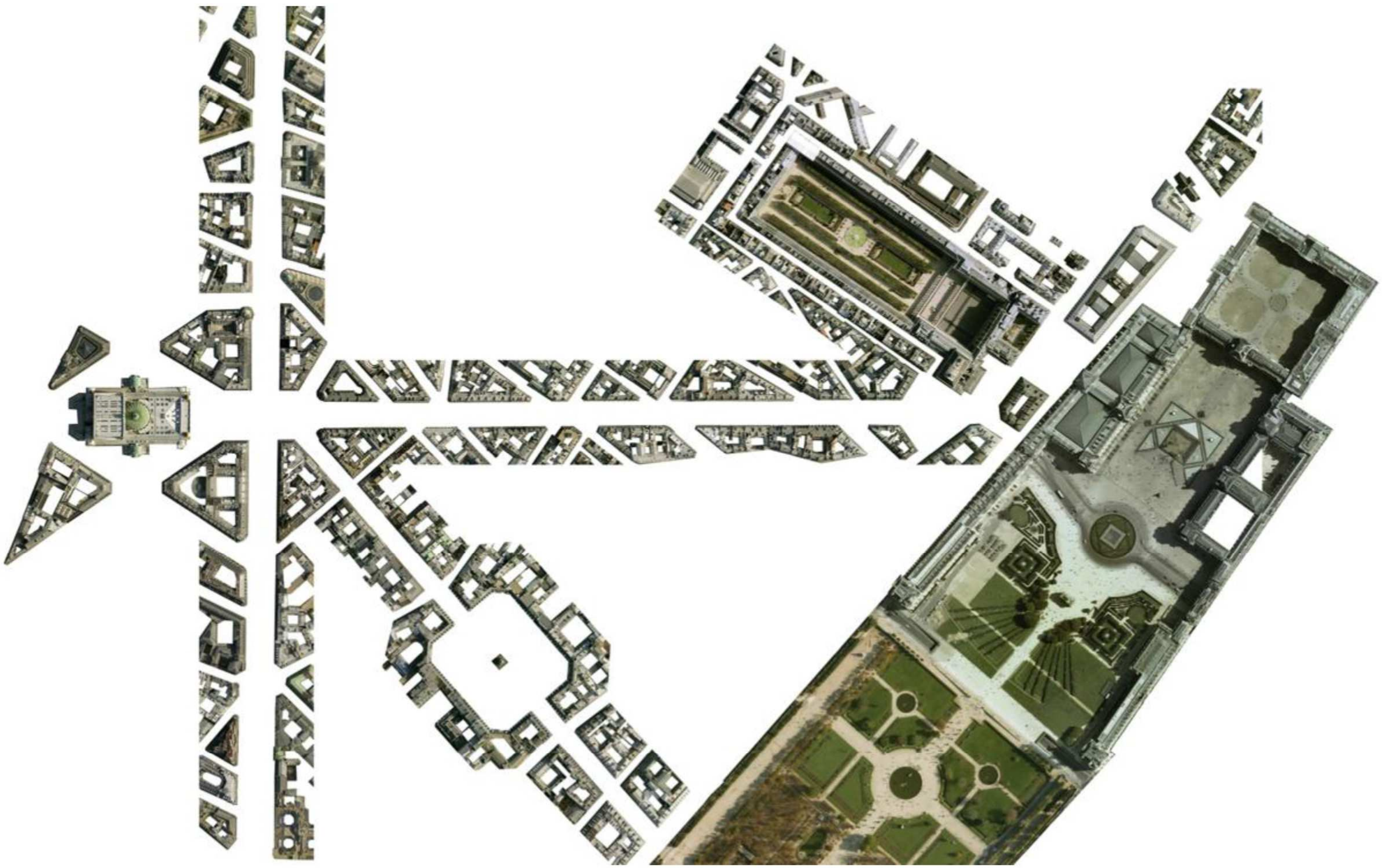
► Plan et vue axonométrique
l'immeubles reconstruits
l'alignement de l'avenue de
Opéra. (Coll. P. Pinon)

The impact of Haussmann intervention on the lot shapes (loss of rectangularity) and the rebuilding of new street frontages (left Avenue de l'Opéra)

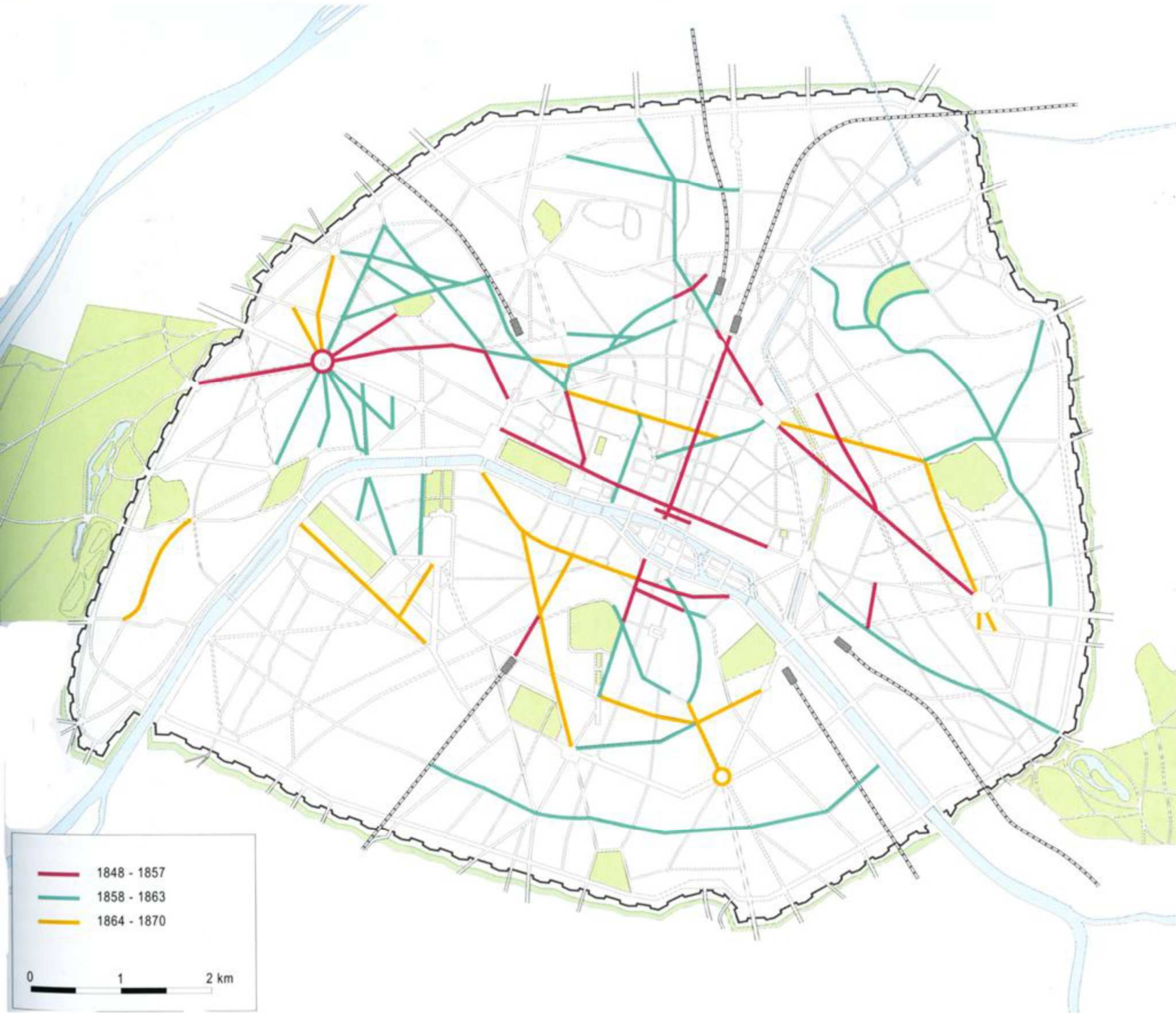
Haussmann's plan implied a large number of destruction and rebuilding: approximately 28, 000 houses were destroyed and 100, 000 were built.

Source: Pierre Pinon, At*las du Paris haussmannien*

Paris: Haussmann cuts into the more than one thousand years old urban fabric. Did it lead to a mathematical order?



Source: Serge Salat, *Cities and Forms*

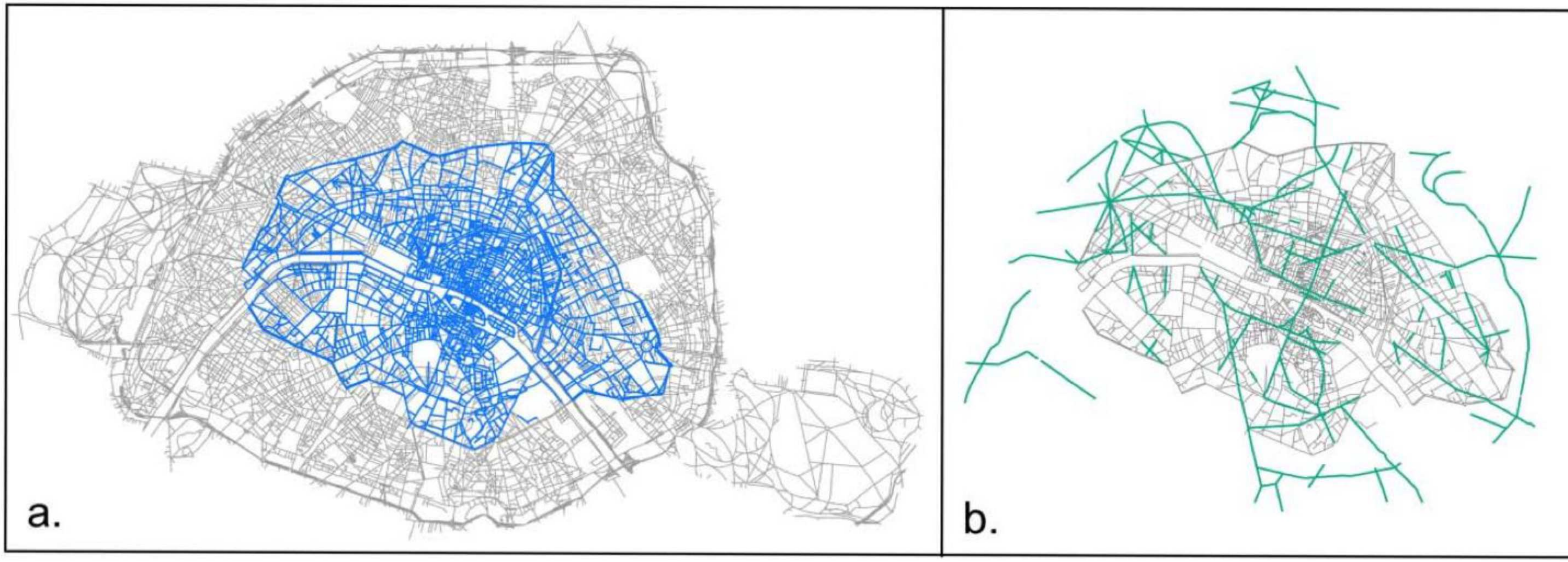


Map of the cuts (« percées) decided between 1848 and 1870

The evolution of Paris results from the superimposition of continuous, local growth processes and punctual changes operating at large spatial scales.
The most important quantitative signatures of Haussmann planning are the spatial reorganization of centrality and the modification of the block shape distribution.

Source: Pierre Pinon, At*las du Paris haussmannien*

1848-1857:The first reordering of the center.
1858-1863: The conquest of the periphery.
1864-1870: The final integration of the center into a higher scale order.

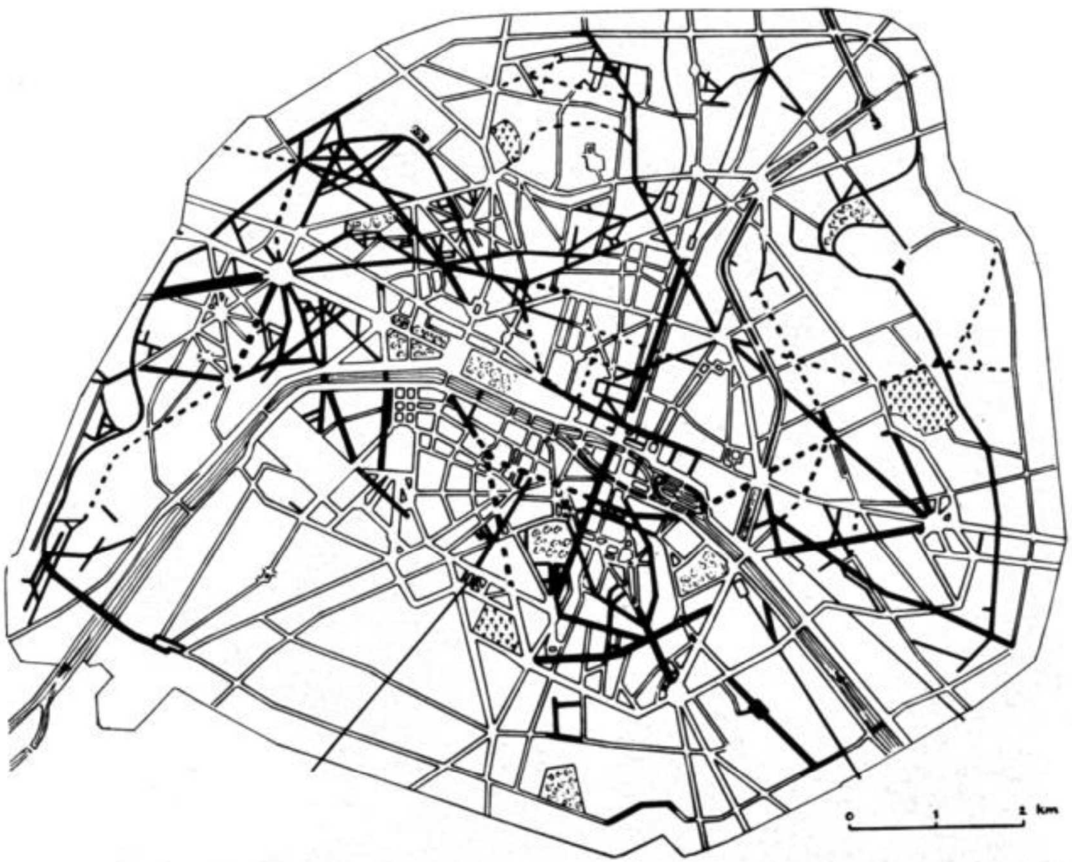


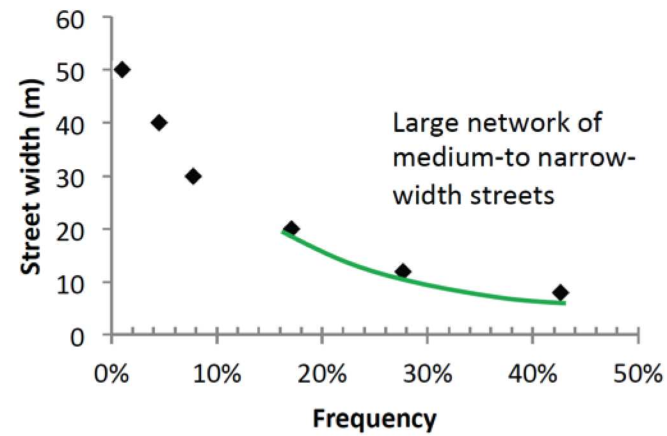
(a) Map of Paris in 1789 superimposed on the map of current 2010 Paris.
(b) Map of Haussmann modifications. The grey lines represent the road network in 1836 (Plan Vasserot), the green lines represent the Haussmann modifications.

In the area corresponding to 1789), the number of nodes of the streets graph increased from about 3000 in 1836 to about 6000 in 1888 and the total length increase from about 400 km to almost 700 km, all this in about 50 years.

Source: Barthelemy, Bordin,Beresticky, Gribaudi,
« Self-organization versus top-down planning in the evolution of a city ».

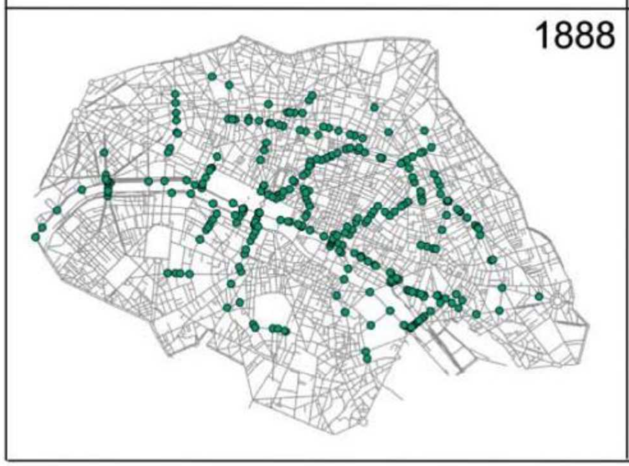
Haussmann reinforced the scaling structure of Paris by integrating the existing city into a larger scale free structure





Scale-free distribution of street widths in Paris

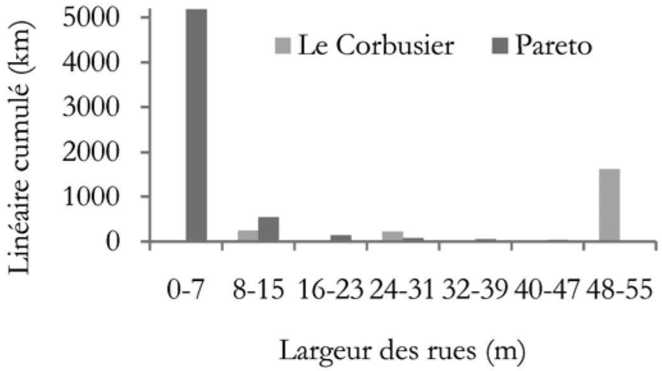
Source: Serge Salat, *Cities and Forms*

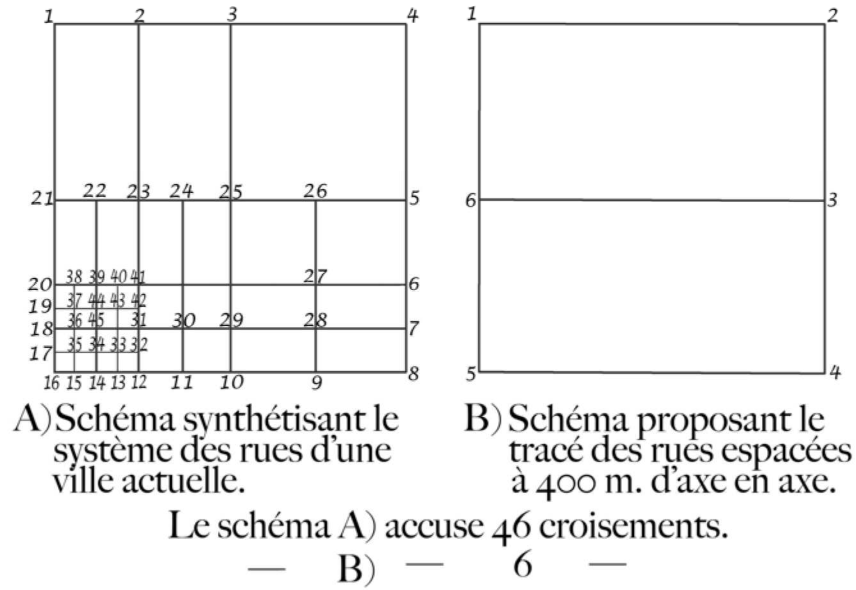


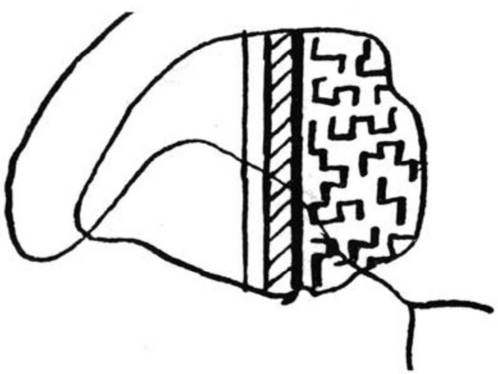
The new large scale order can be measured in the dramatic spatial reorganization of betweenness centrality

The spatial distribution of the BC has not been stable in Paris during the 19th century. It displays large variations, and is not uniformly distributed over the Paris area . Between 1836 and 1888, Haussmann works had a dramatic impact on the spatial structure of the centrality, especially near the heart of Paris. Central roads usually persist in time, but Haussmann reorganization was acting precisely at this level by redistributing the shortest paths.

Source: Barthelemy, Bordin, Beresticky, Gribaudi, « Self-organization versus top-down planning in the evolution of a city ».



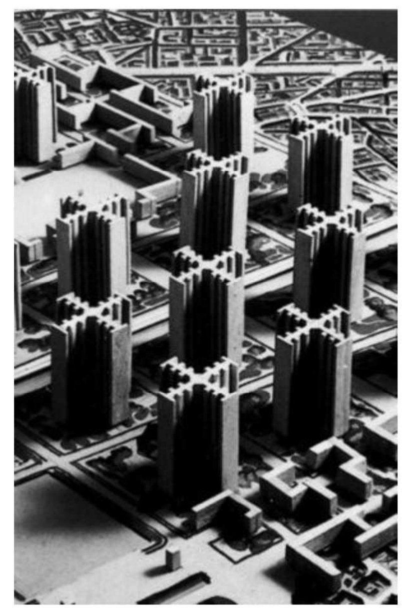




Le Corbusier's schematic drawings for eliminating all the subscales of the urban network

It ends up in an inverted power law pattern

Source: Serge Salat, *Cities and Forms and* Le Corbusier *Urbanisme*

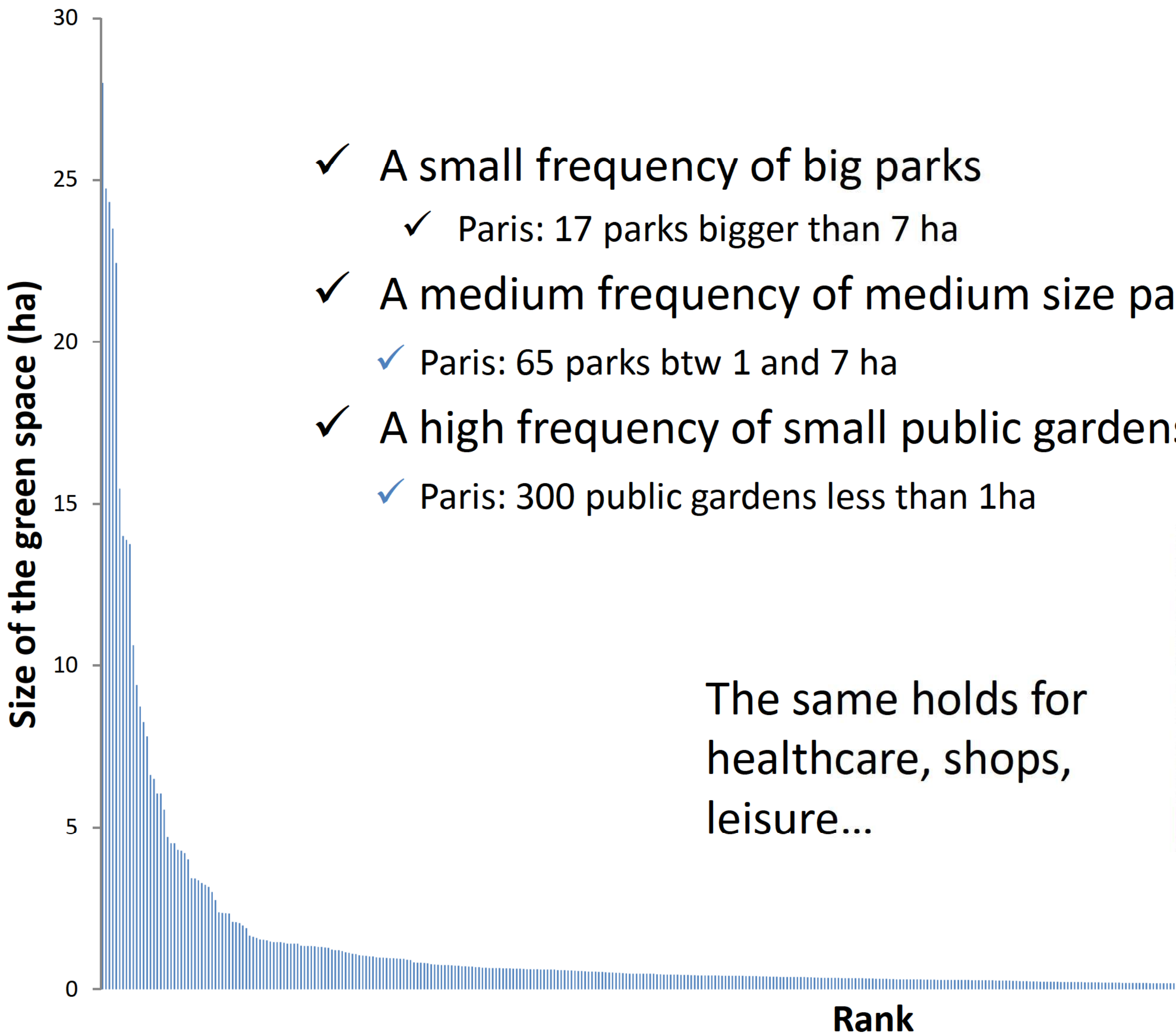


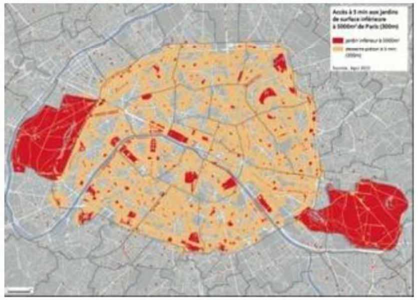
At intra urban scales, accessibility is enhanced by a scale-free distribution of amenities within the urban fabric

APUR

In Paris intra-muros, scale free distributions enhance accessibility with a long tail of small elements

Source: Urban Morphology Institute



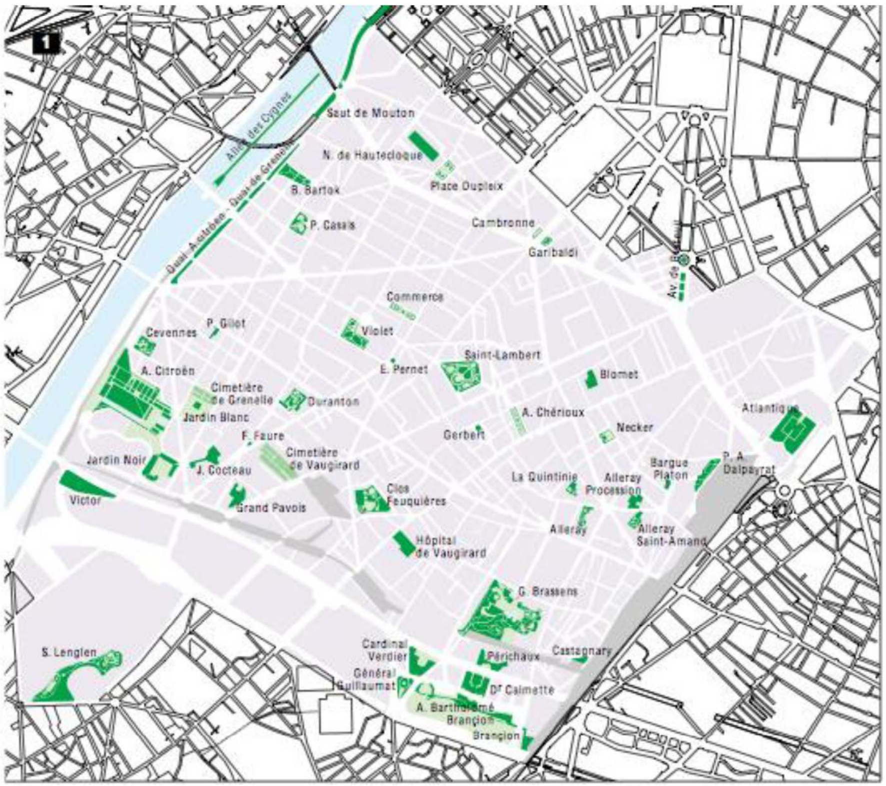


- ✓ A small frequency of big parks
  - ✓ Paris: 17 parks bigger than 7 ha
- ✓ A medium frequency of medium size parks
  - ✓ Paris: 65 parks btw 1 and 7 ha
- ✓ A high frequency of small public gardens
  - ✓ Paris: 300 public gardens less than 1ha

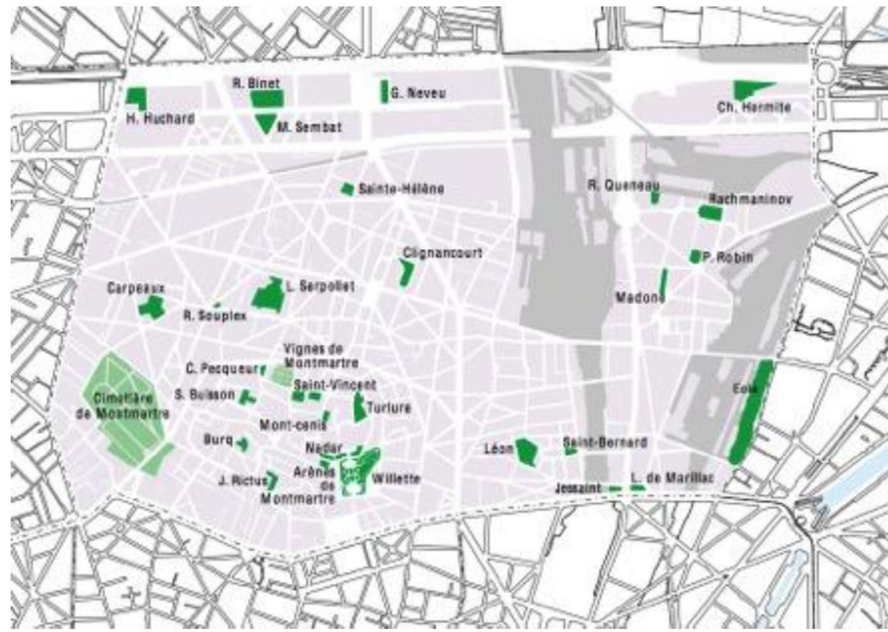
The same holds for healthcare, shops, leisure…

The inverse power law distribution distribution of sizes is verified at arrondissement scale

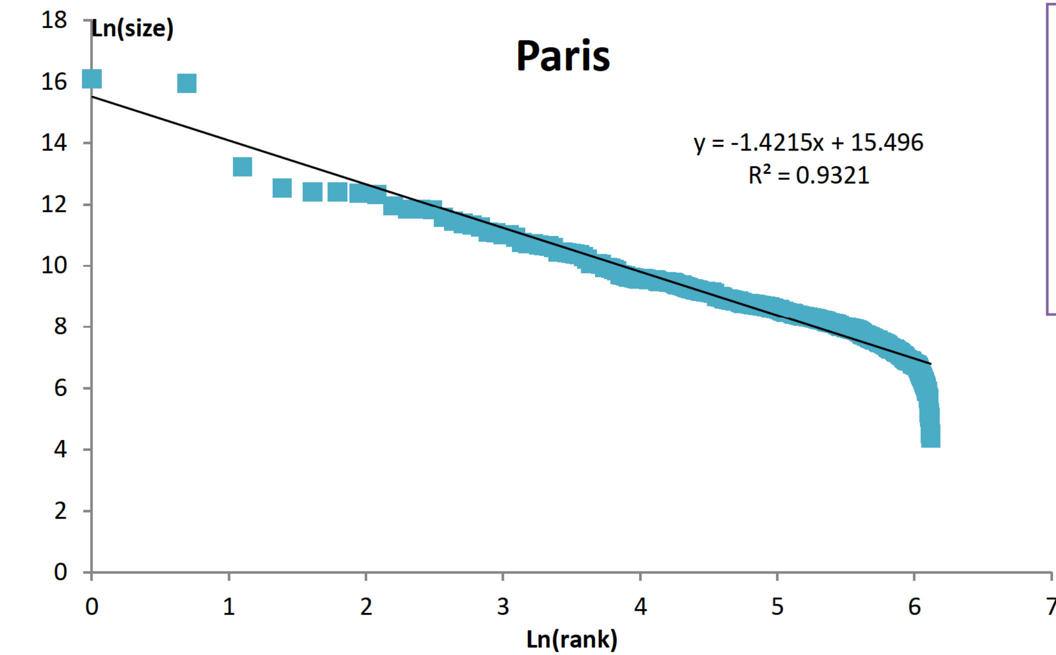
15ème arrondissement



18ème arrondissement

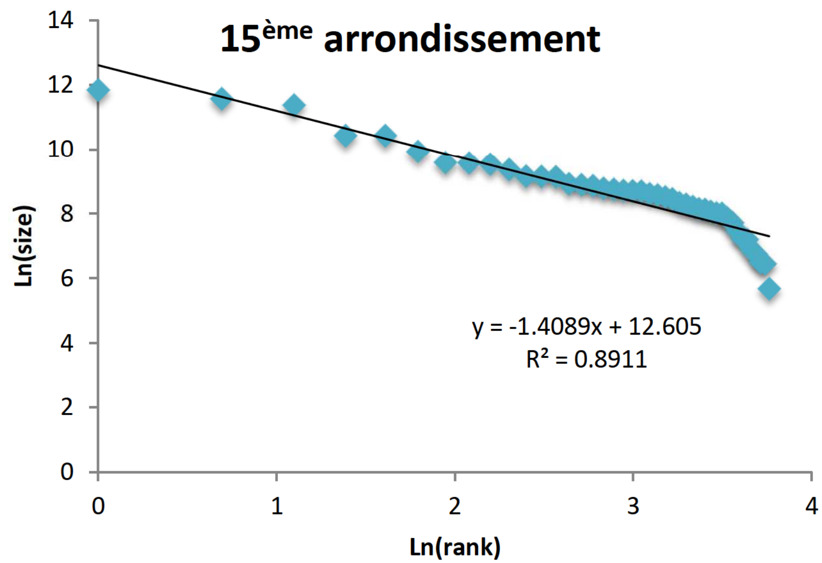


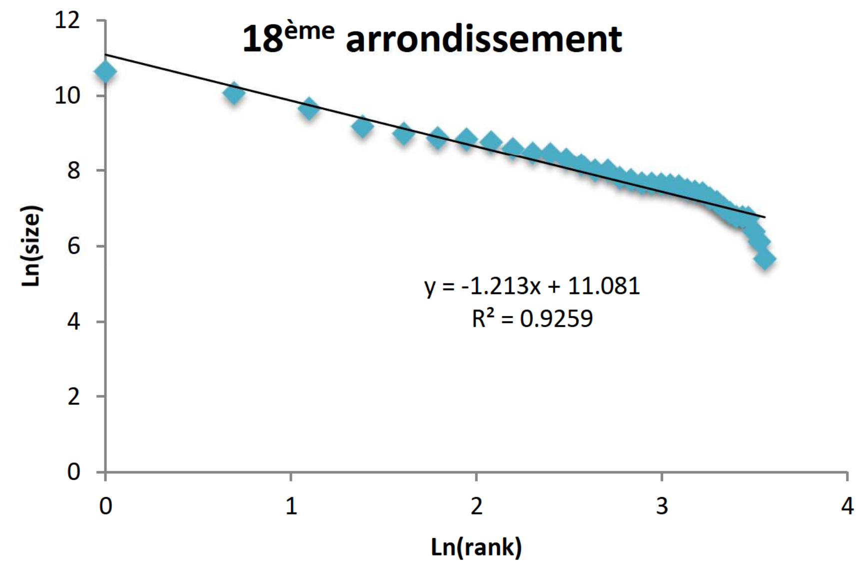
Source : APUR



Public parks distribution in Paris intra-muros follows an inverse power law. The same holds at arrondissement scale. The distribution is scale free

Source: Urban Morphology Institute

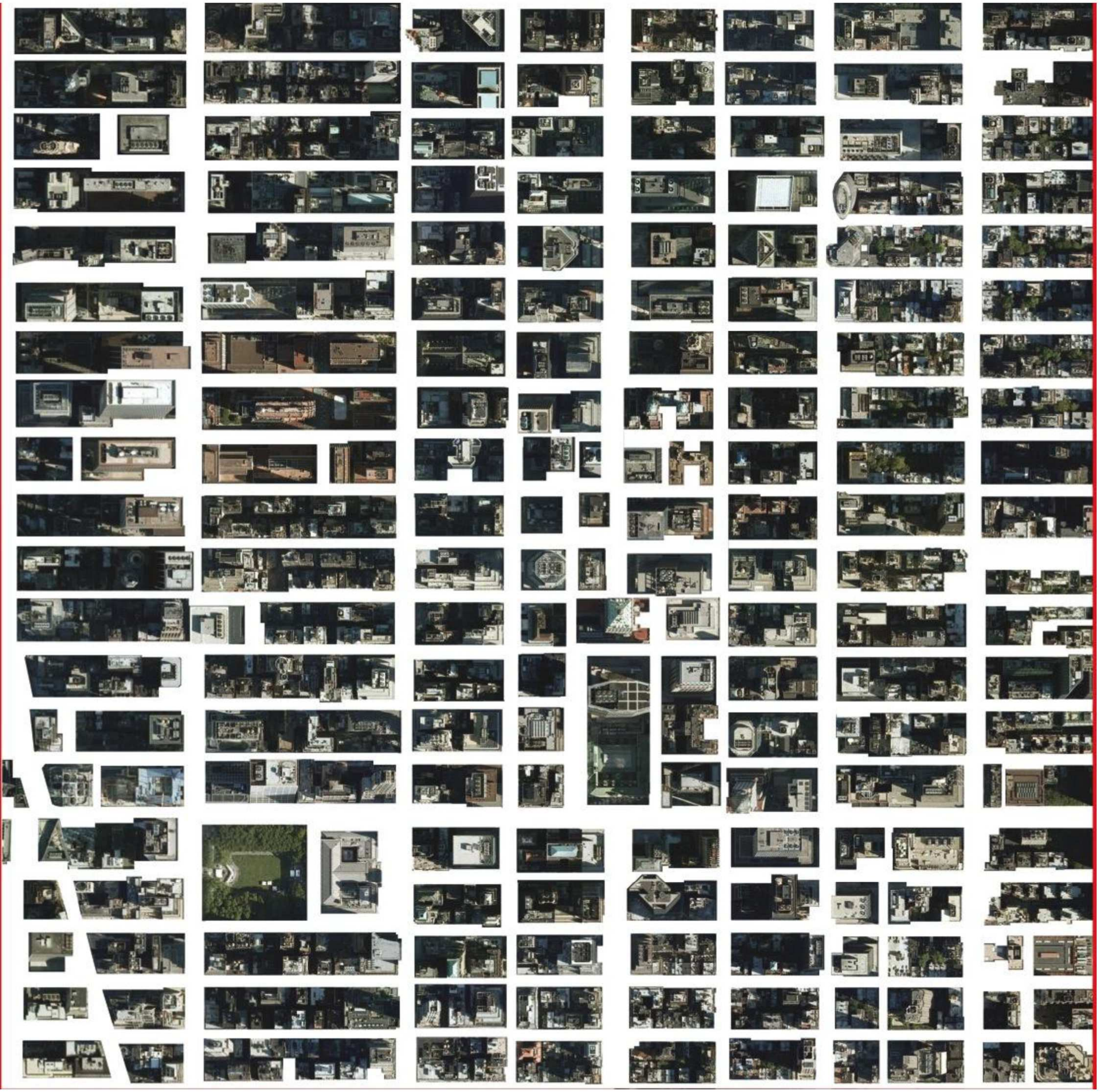




New York

Fine grain urban microstructure consolidates overtime in a multifractal structure

The demography, energy and economic urban landscape is extremely bumpy at all scales even within a regular Euclidean grid



1 square mile of Manhattan

A multifractal structure within an Euclidean grid, shaped by 200 years of market forces

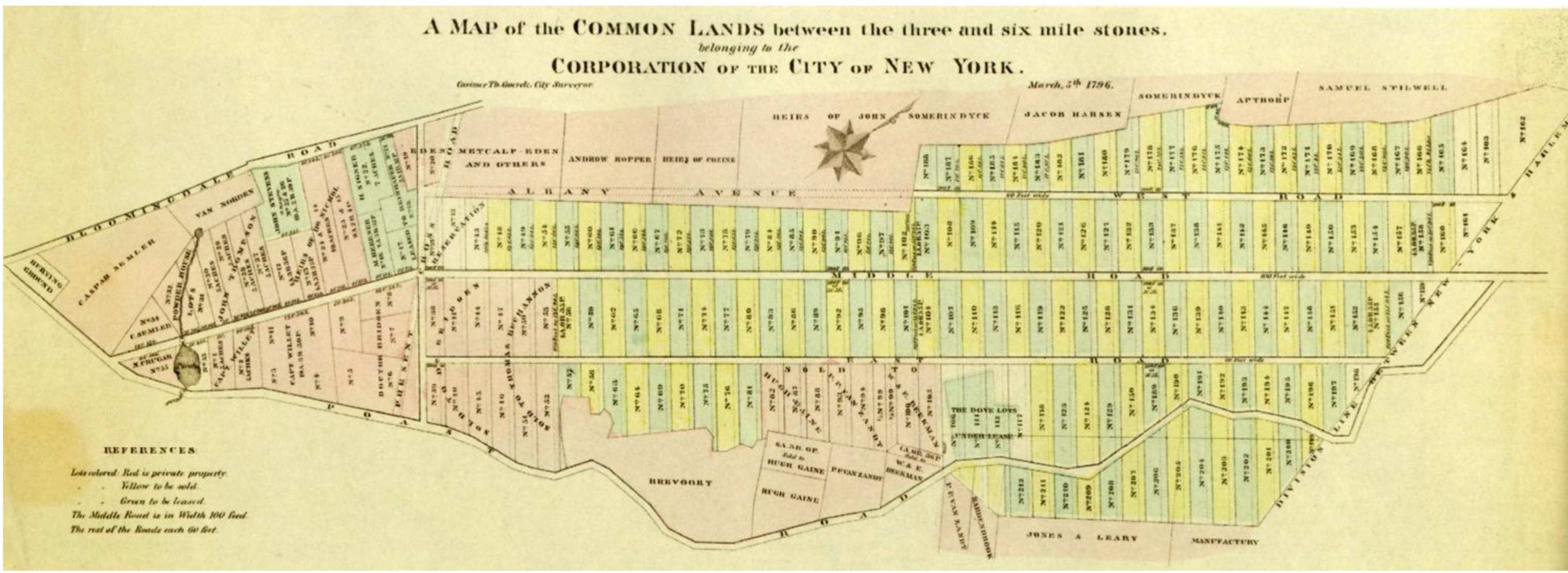
Source: Serge Salat, *Cities and Forms*

The resilience of street patterns
1 square mile of Lower Manhattan in 1776 and now

Source: Serge Salat, *Cities and Forms*

The Goerck plan of the Common lands 1796

The Common lands were vacant land first granted by Dutch provincial authority to the government of New Amsterdam in 1658. After the American Revolution, the new and cash-strapped American city government looked to profit from its underperforming domain (about 2 square miles of rocky, hilly undesirable land in the middle of the island.
To facilitate sale of the Common lands, Goerk prepared a subdivision plan with 3 long parallel streets, which would become 4th, 5th and 6th avenues, with an east-west length of blocks identical to the one in Goerck's plan. This plan started the rise of NY real estate market and ascent of land values.



The Commissioners' map of 1807 overlays a seemingly uniform grid of rectangles over the rugged island.

In reality the grid contains **2 patterns** that create variety.

- One pattern is formed by the street widths (100 feet for the avenues, 60 feet for standard cross streets, with 15 major cross streets 100 feet at irregular intervals.

- The second pattern derives from block dimensions. All blocks are 200 feet wide north to south, but their length east to west varies diminishing from the center to the shorelines. From Third to Sixth avenue blocks are 920 feet long. Moving eastward they shrink 620, 650, 640 feet long. Moving westward, they shrink uniformly to 800 feet long

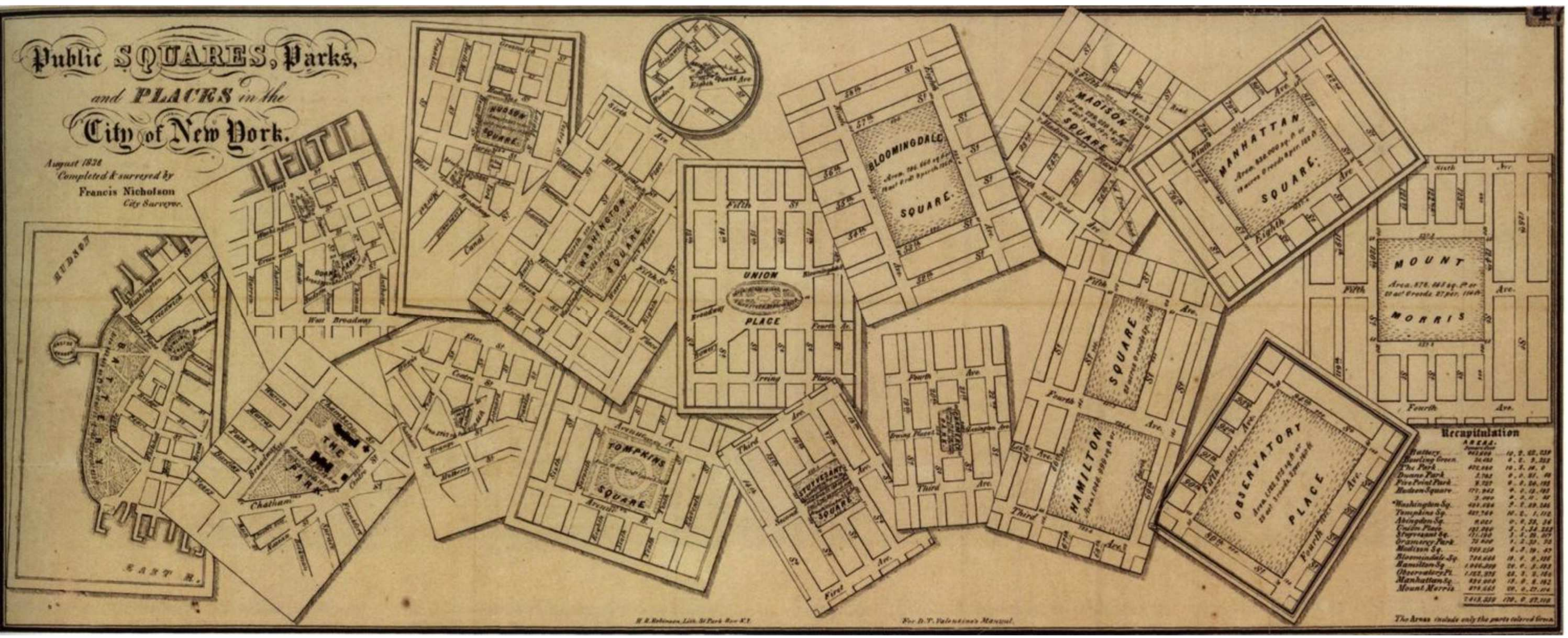
Elgin Garden
Kip's Bay
Bellevue
PARADE
Powder House
MARKET PLACE
Sailors snug harbor
Townsend
Clinton
Second Avenue
First Avenue
Avenue
Jones St
Bond St
Stuyvesant
Street
Brown's Wharf

The Bridges Map 1811

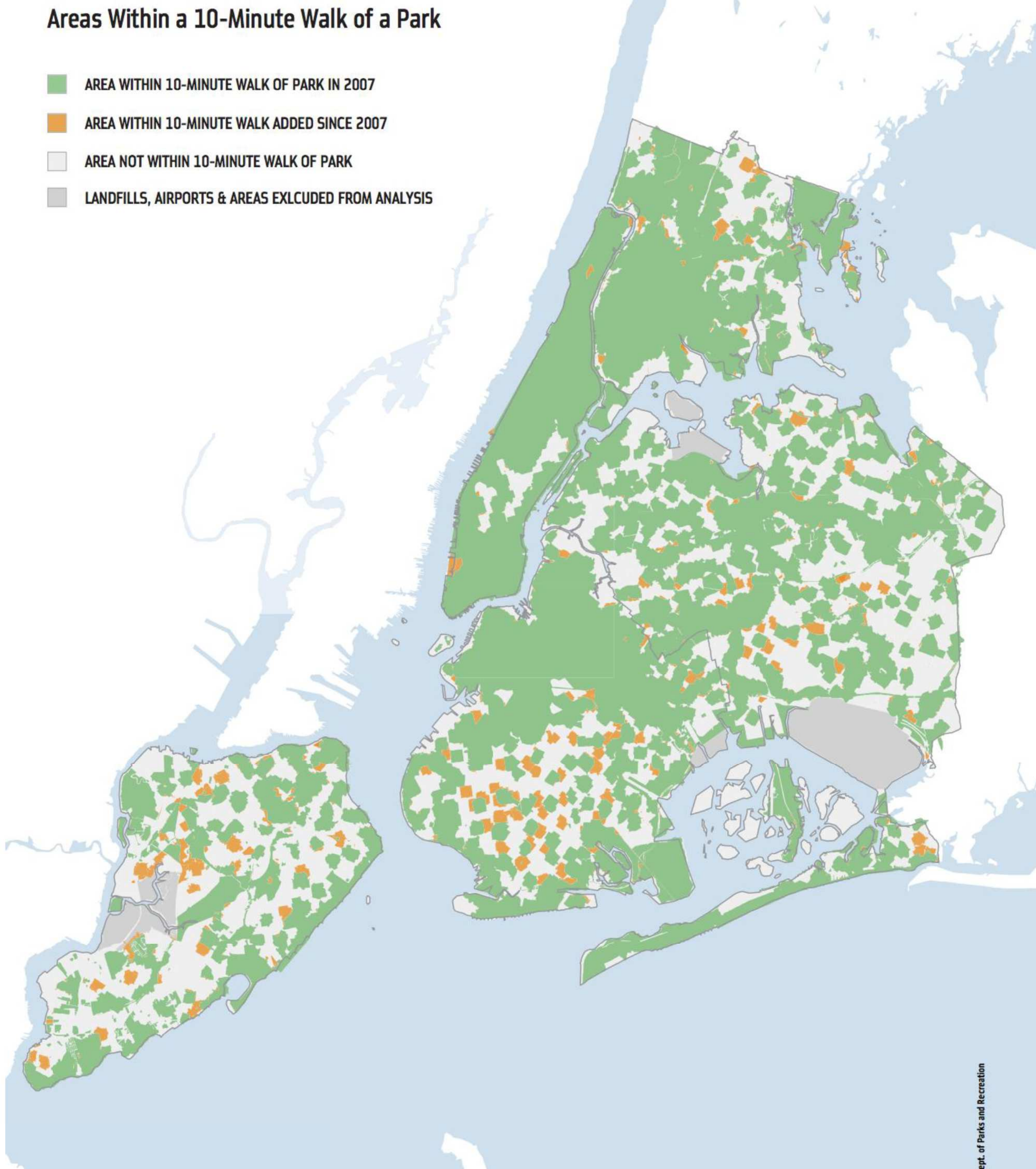
In this pictorial copy of the Commissioners' map of 1811, note the large gardens and the Parade that have been replaced quickly by a policy of more numerous smaller gardens in order to increase land value.

New York Manhattan Collage of public parks 1838

Instead of a few large parks as envisioned in the Commissioners' plan, the city's growing reliance on real estate taxes motivated officials to improve property values by opening parks as a means to collect more money for the municipality.
In 1830, property tax revenue amounted to roughly \$200,000, but 7 years later, they totaled \$ 1.1 million.

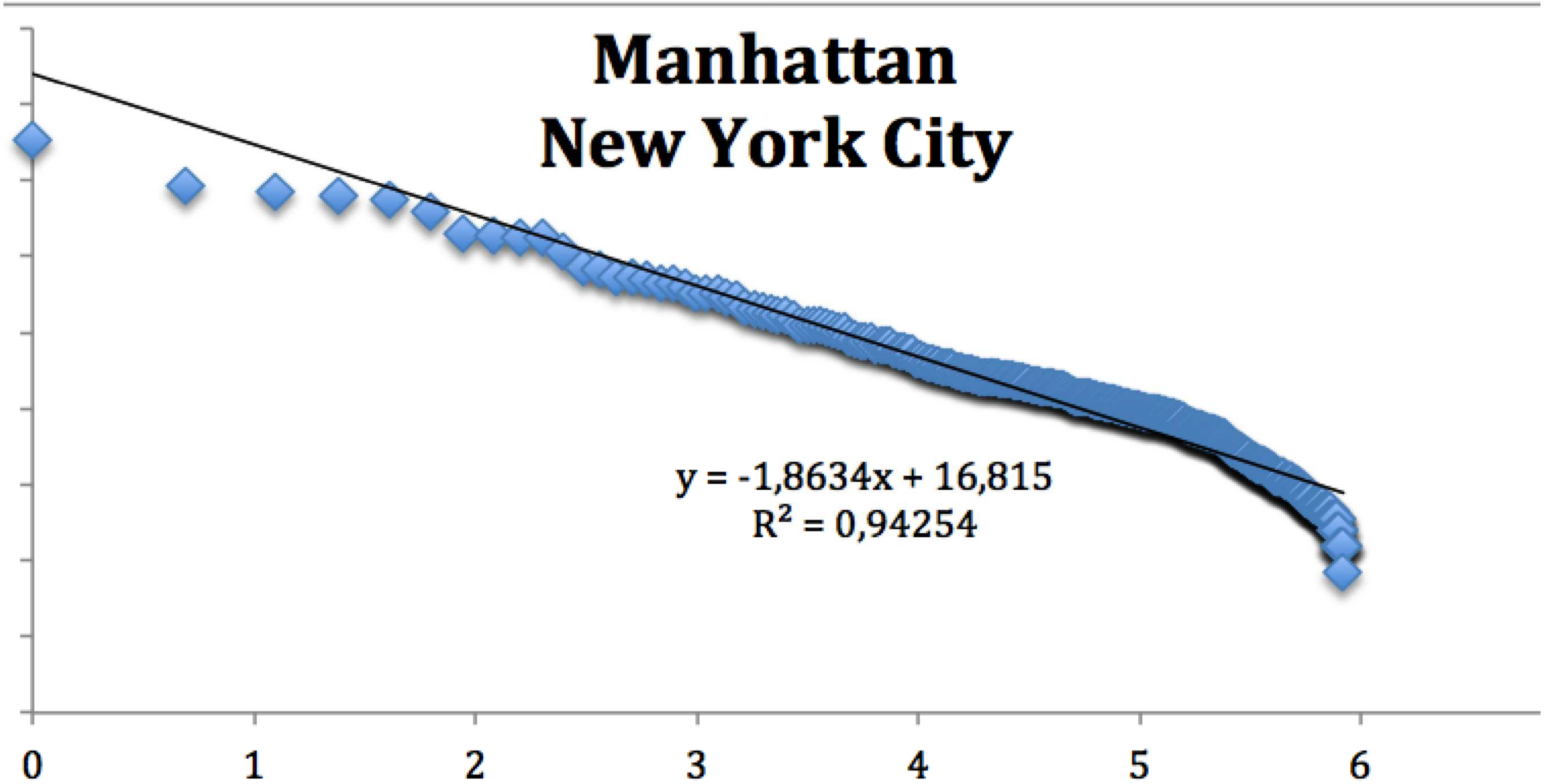


As a result of early 19th century real estate speculation, Manhattan island accessibility to public parks is optimal today

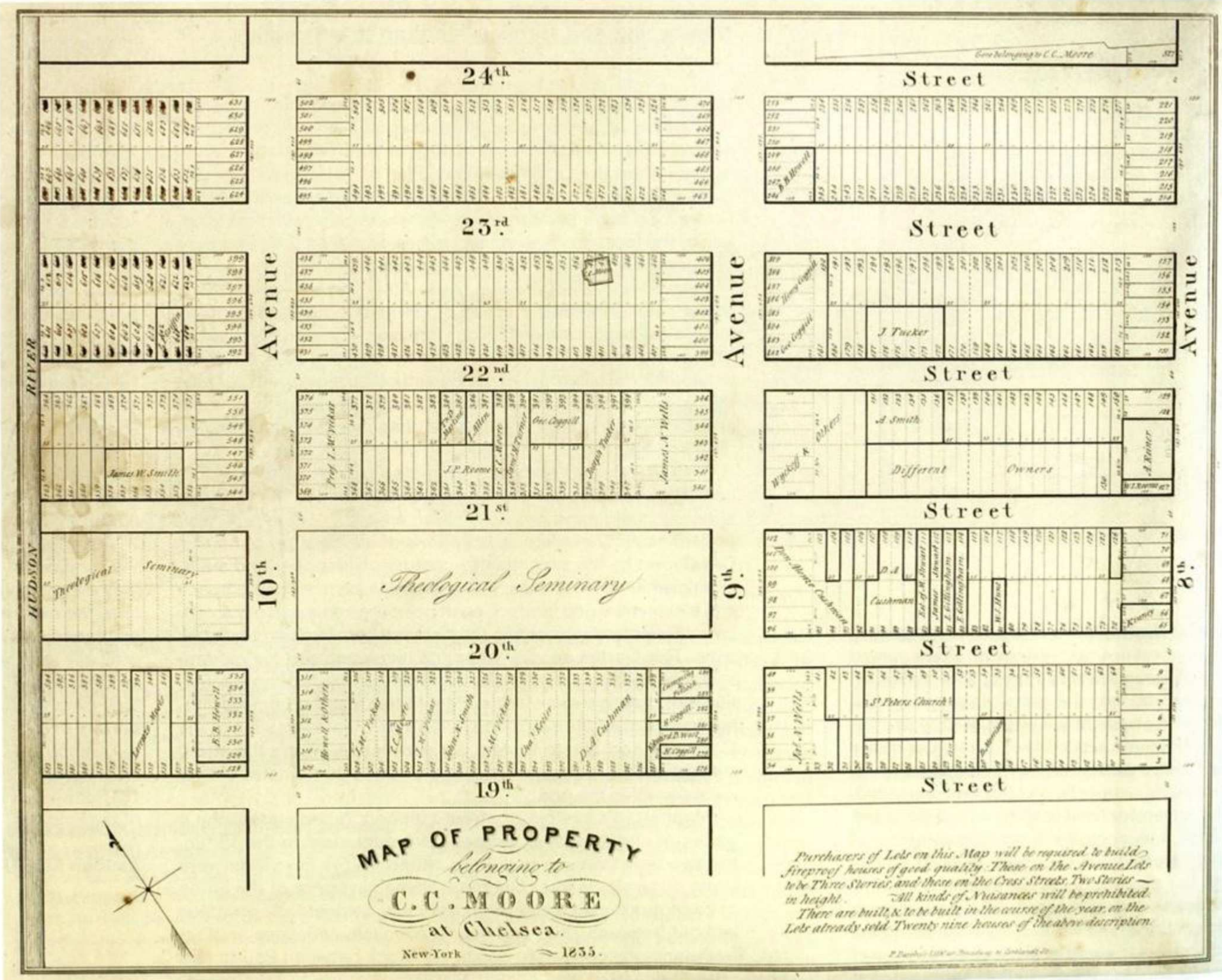


Source: PlaNYC

The sizes of Public parks in Manhattan follow an inverse power law with a hierarchy exponent higher than in Paris



Source: Urban Morphology Institute

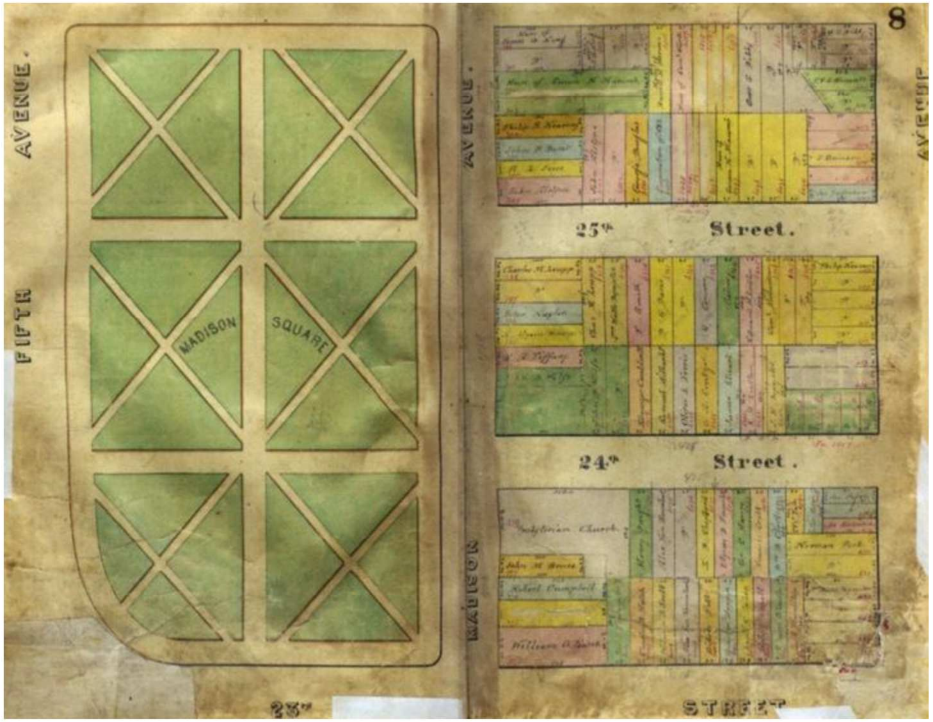


From the property of Clement Clarke Moore to Chelsea neighborhood

Free market is a formidable time accelerator for differentiation and emergence of scale free structures. A early as 1835, this map illustrates the strategies of sellers of vast estates. Moore developed his estate into Chelsea village centered around Chelsea Square he had donated to the episcopal church in 1819. Note the early differentiation of lots

X 35

In 1820 Moore had evaluated his estate at $ 17,000. His wealth was estimated at $ 350,000 in 1845 and $ 600,000 in 1855.



Assessment map of Madison square
1853 - 1879

Differentiation and asymmetry in land prices occurred very quickly in the seemingly uniform Manhattan grid

In 1860, real estate along Fourth Avenue in the section depicted ranged from \$ 3,500 to \$ 8,000, while lots along Madison avenue were valued between \$ 18,000 and 55,000.

X 8 population increase in 50 years

X 80 real estate value in 80 years

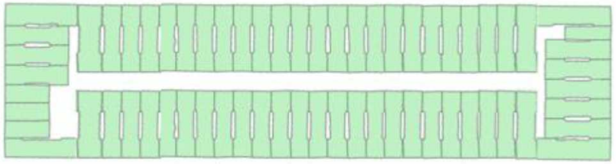
Between 1790 and 1810, Manhattan population tripled to 96,000 inhabitants. The Commissioner's plan envisioned a scenario reaching 155th Street with 400,000 in 1860. In 1810, Peking and London each had over one million residents and Paris half million. In 1860, NY population was actually 813,000, doubling the projection.

The Grid was above all an easy format for the subdivision and development of land. The grid system stripped the land of topographical markers and specificity, and repackaged it as standardized building lots. The grid reconceptualized the island in a real estate market. And it worked beyond all expectations. In 1807, the assessed value of New York City real estate was $ 25 million. In 1887 it was $2 billion, a 80-fold increase.

Scaling laws in platting are the result of evolution when the city is shaped by market forces
The scaling coefficient reflects morphological periods

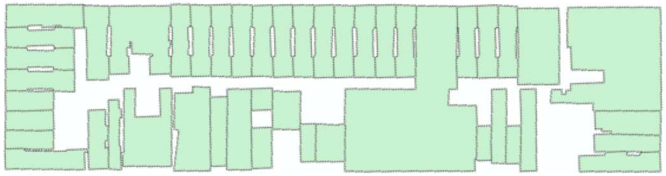
Manhattan original plot subdivision is identical to a 12th century French South West « bastide » (new town). The north-south width of blocks (60 m) is shorter than the Roman empire block size (70 m). Along Fifth Avenue Mahattan street rhythm is Medieval!

Manhattan
Original plot subdivision in 1811
Average plot size: 205 m²



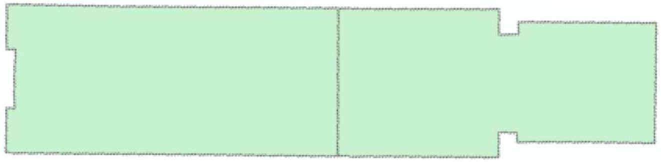
Residential

Manhattan
Intermediary plot consolidation
Average plot size: 255 m²



Mixed use

Manhattan
Extreme plot consolidation
Average plot size: 6,100 m²



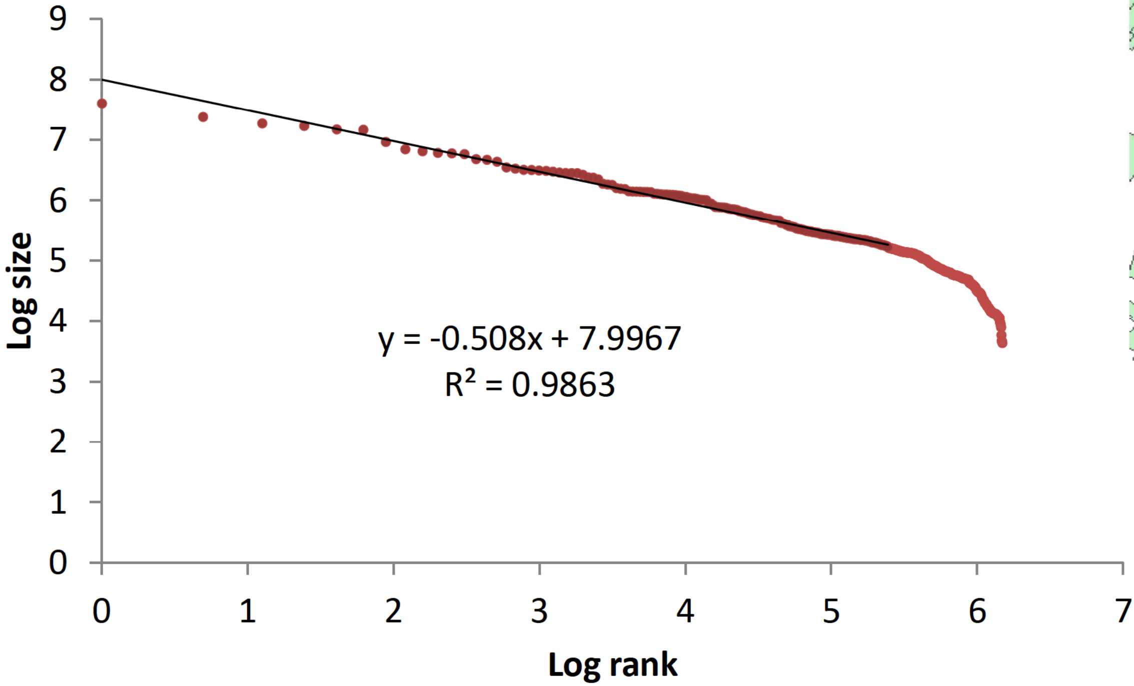
Large businesses

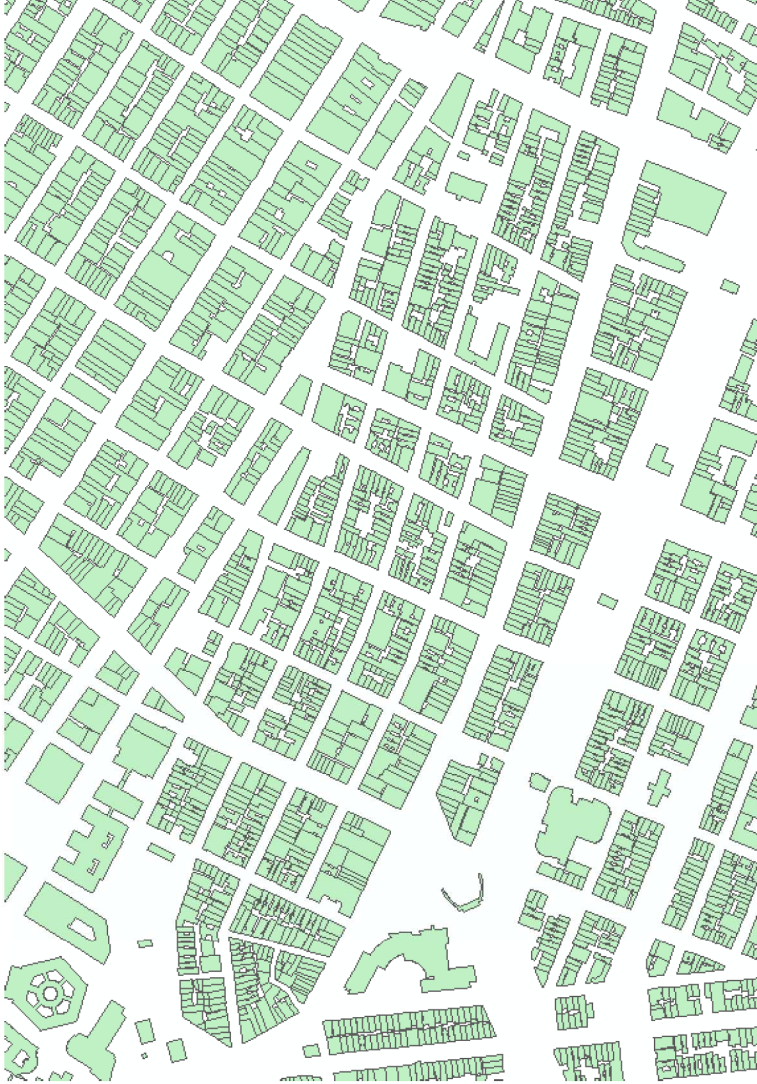
Source: Urban Morphology Institute

Highly adaptive platting follows a mathematical regularity characteristic of scale free complex systems: Frequency of sizes follows an inverse power law

Wall Street's plot area scaling coefficient is similar to Paris reflecting the European origin of this part of the city (New Amsterdam) and its longer evolution The largest plot is 2000 m2.

New York City
Wall Street

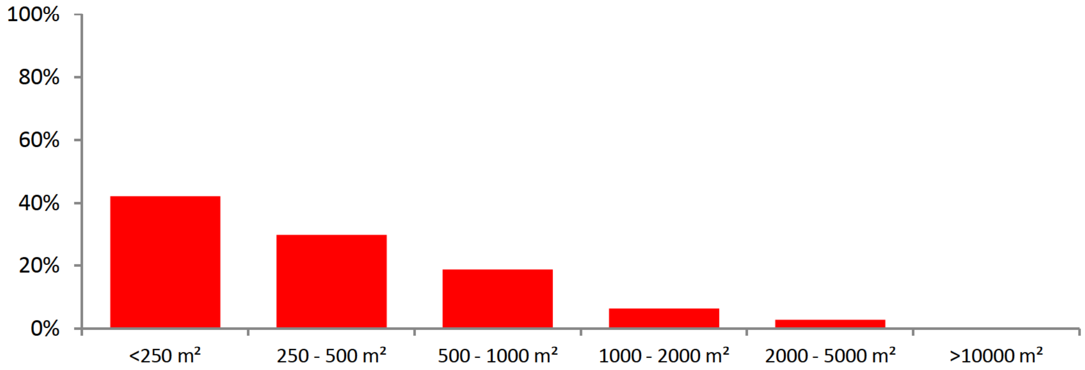




Source: Urban Morphology Institute

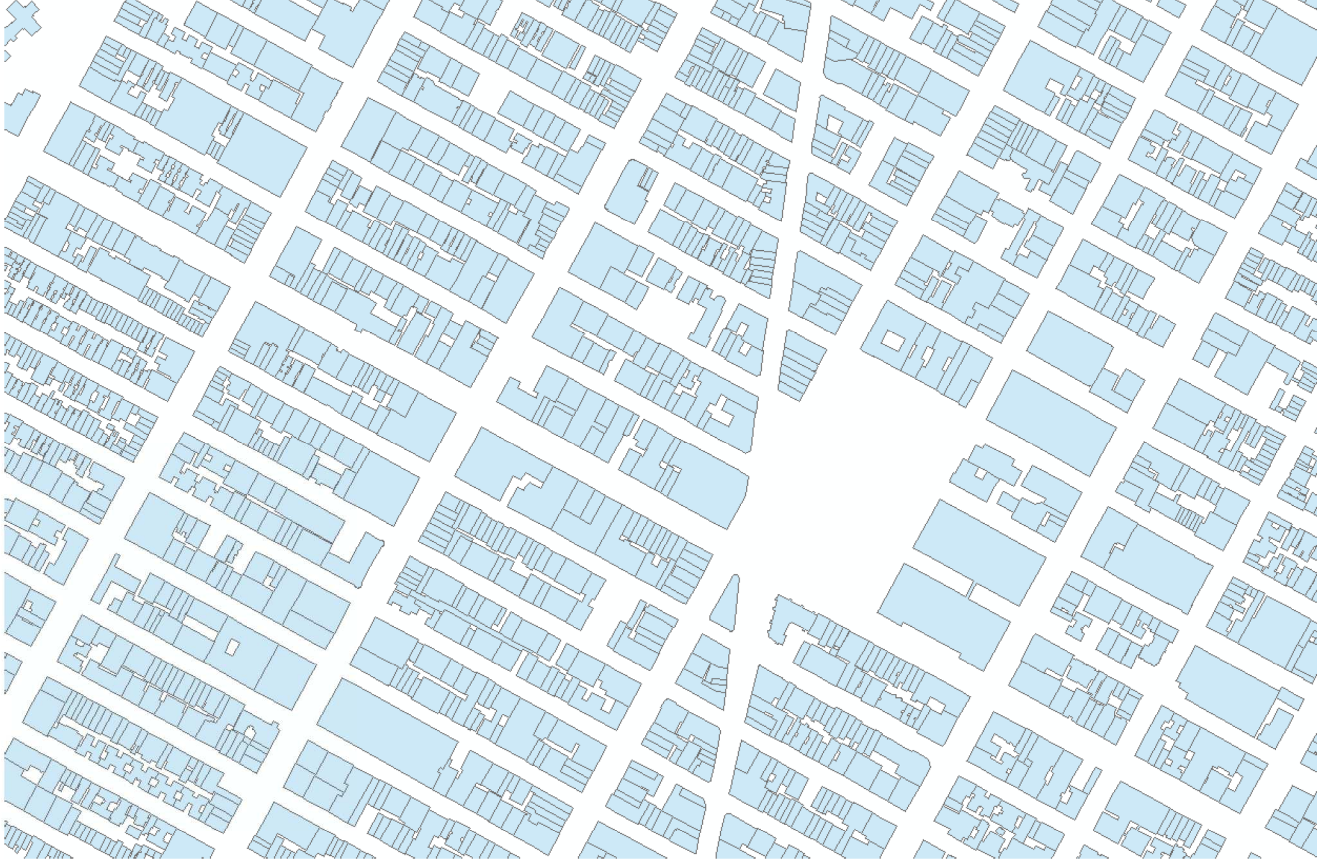
Manhattan
Madison square area

The largest plot is 4,700 m2 reflecting the change of scale in development compared to New Amsterdam.
Only 40% of the plots correspond to the original platting size

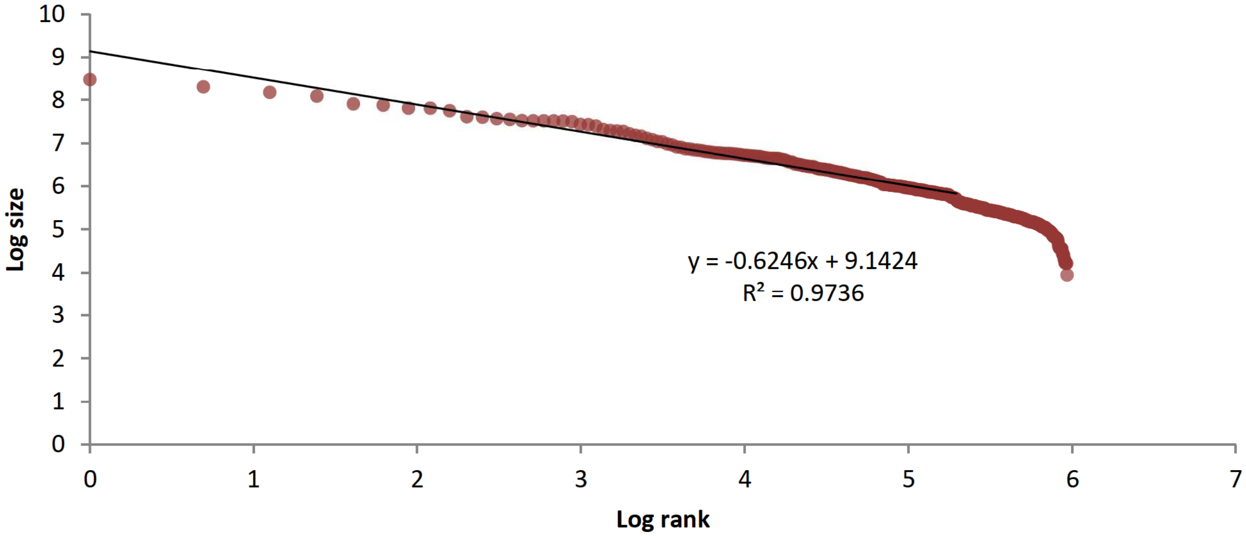


Source: Urban Morphology Institute

Madison Square area has a higher scaling coefficient than Paris or Wall Street.



Manhattan
Madison Square area

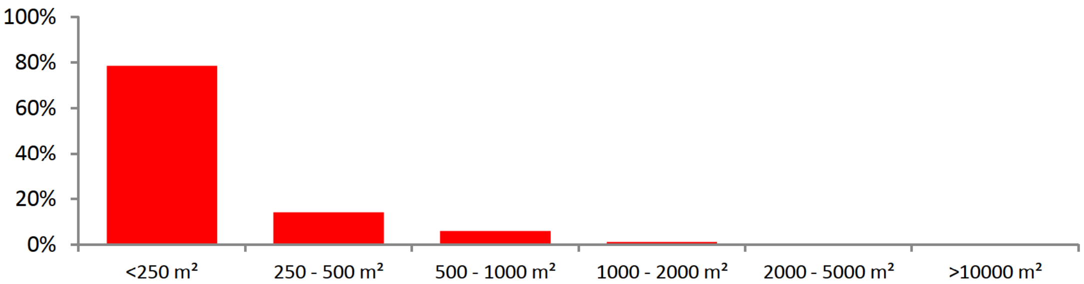


Source: Urban Morphology Institute

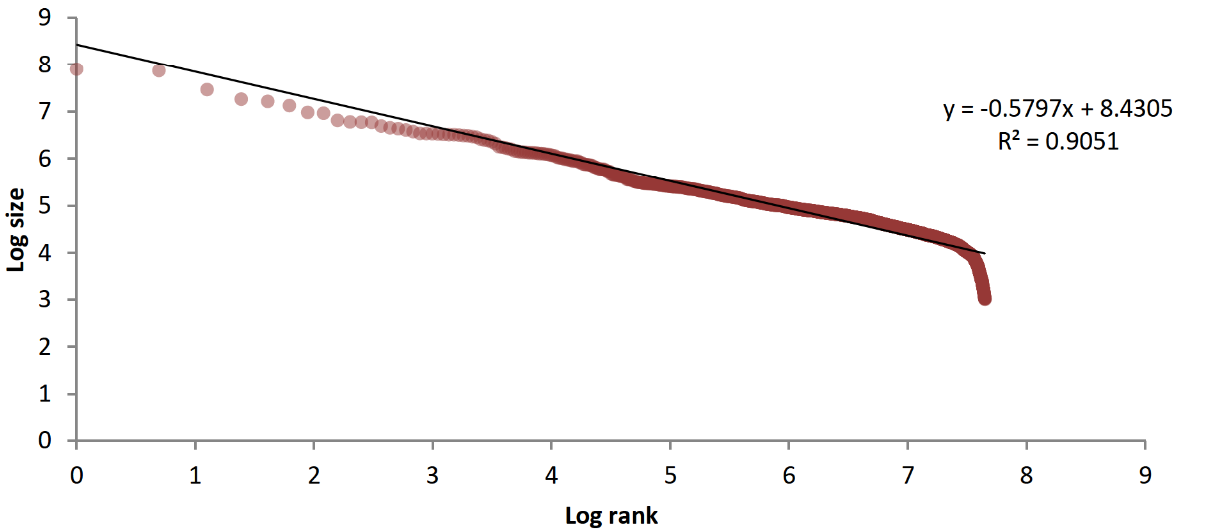
New York City
Brooklyn

More residential Brooklyn shows more of the original platting (80% of the plots) with a scaling coefficient similar to Madison Square. New York signature ?

The largest plot is 2,700 m2



Source: Urban Morphology Institute

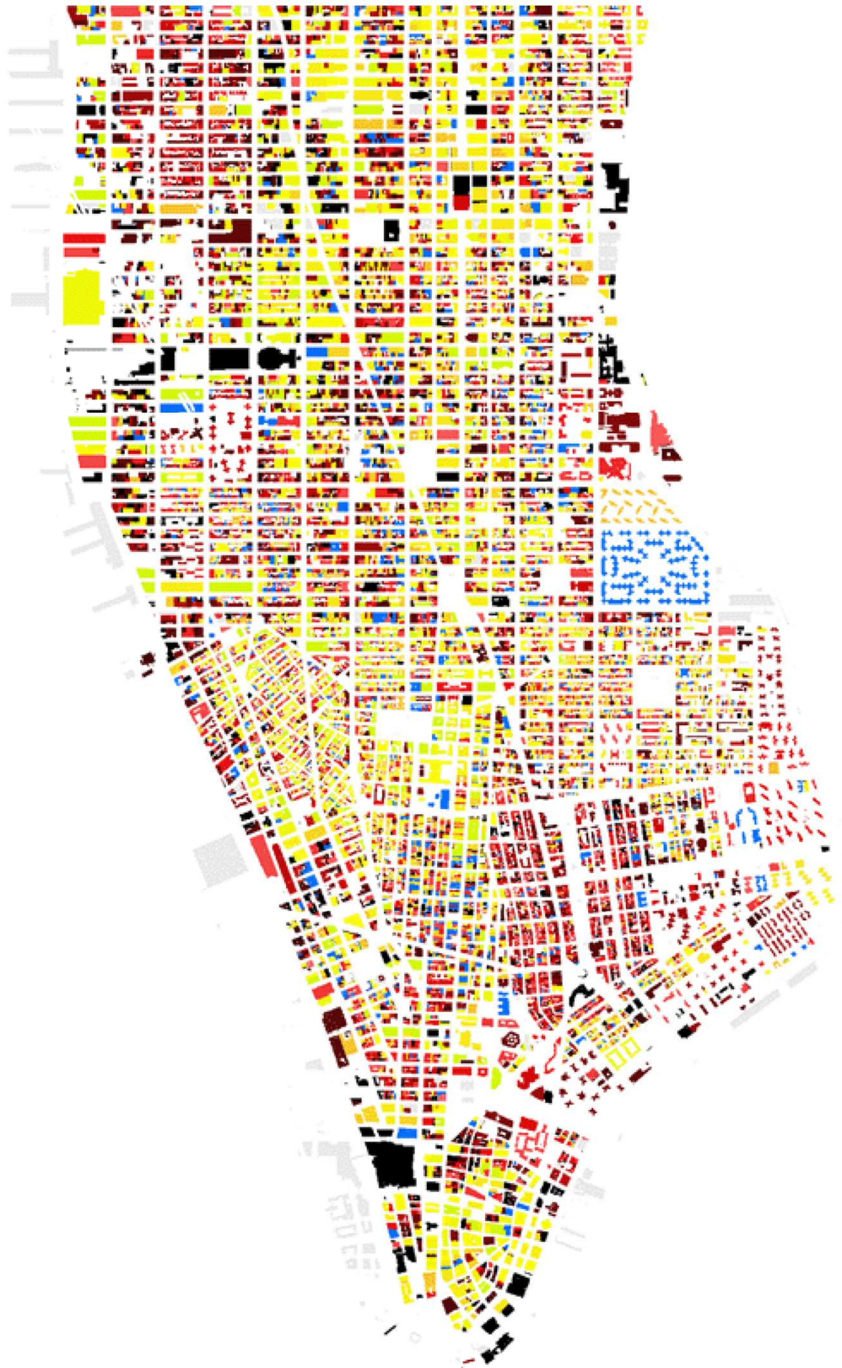


New York City
Brooklyn

Source: Urban Morphology Institute

Manhattan

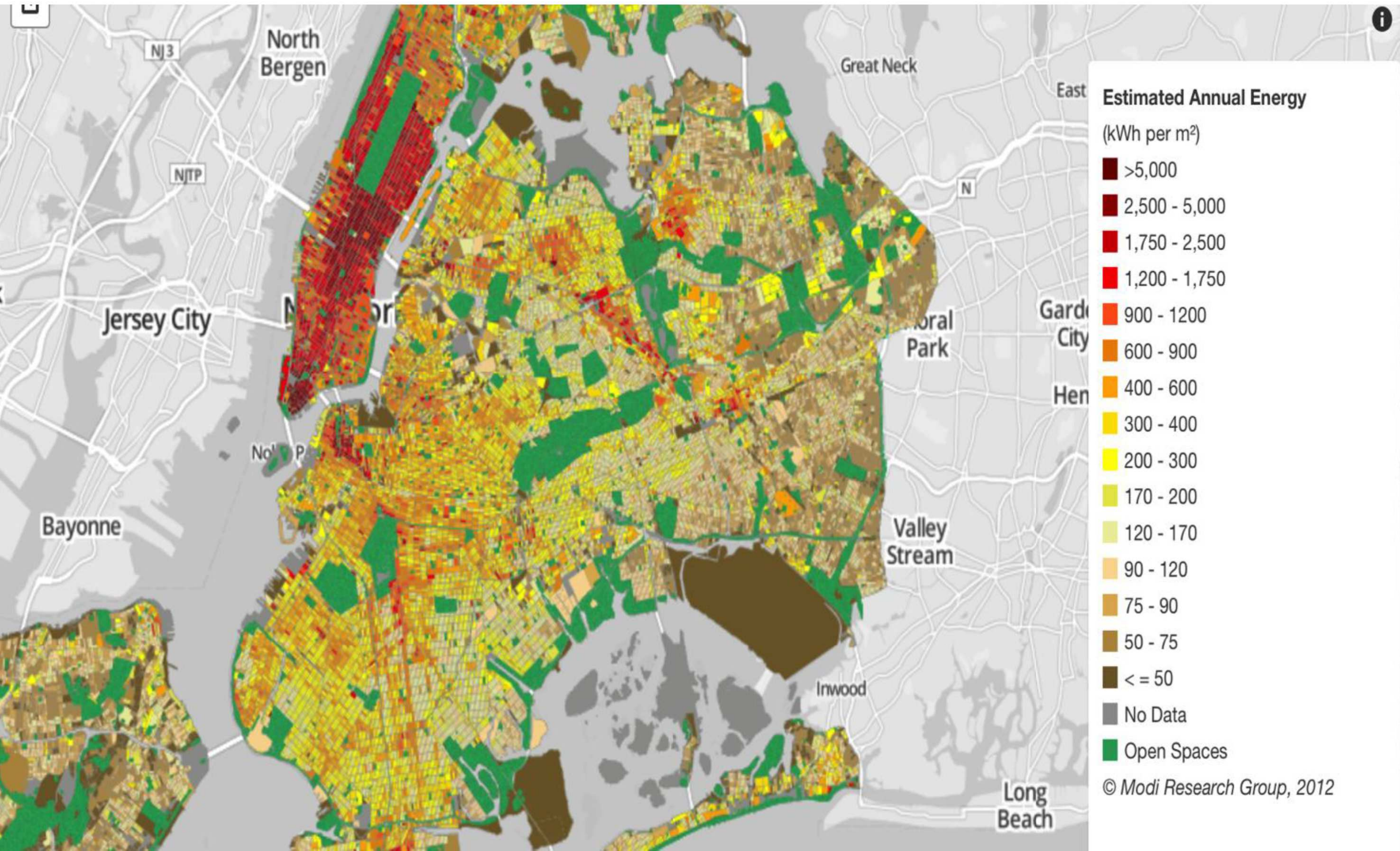
A bumpy multifractal urban landscape within an apparently homogeneous Euclidean grid



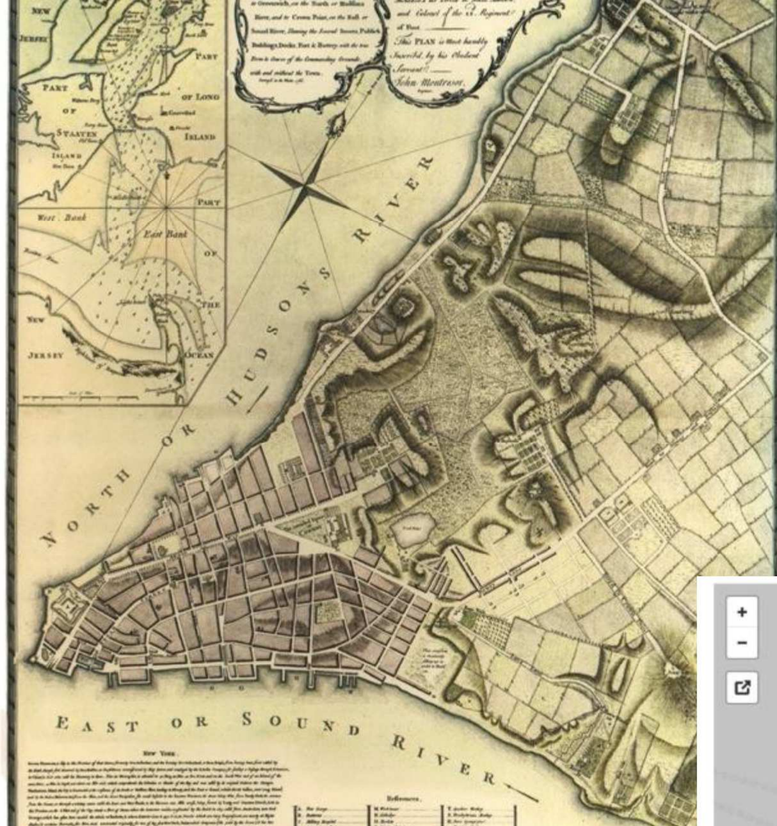
In a “ville pêle-mêle” like New York, the energy density at the tax lot level (in kWh/m²) varies more than 100-fold.

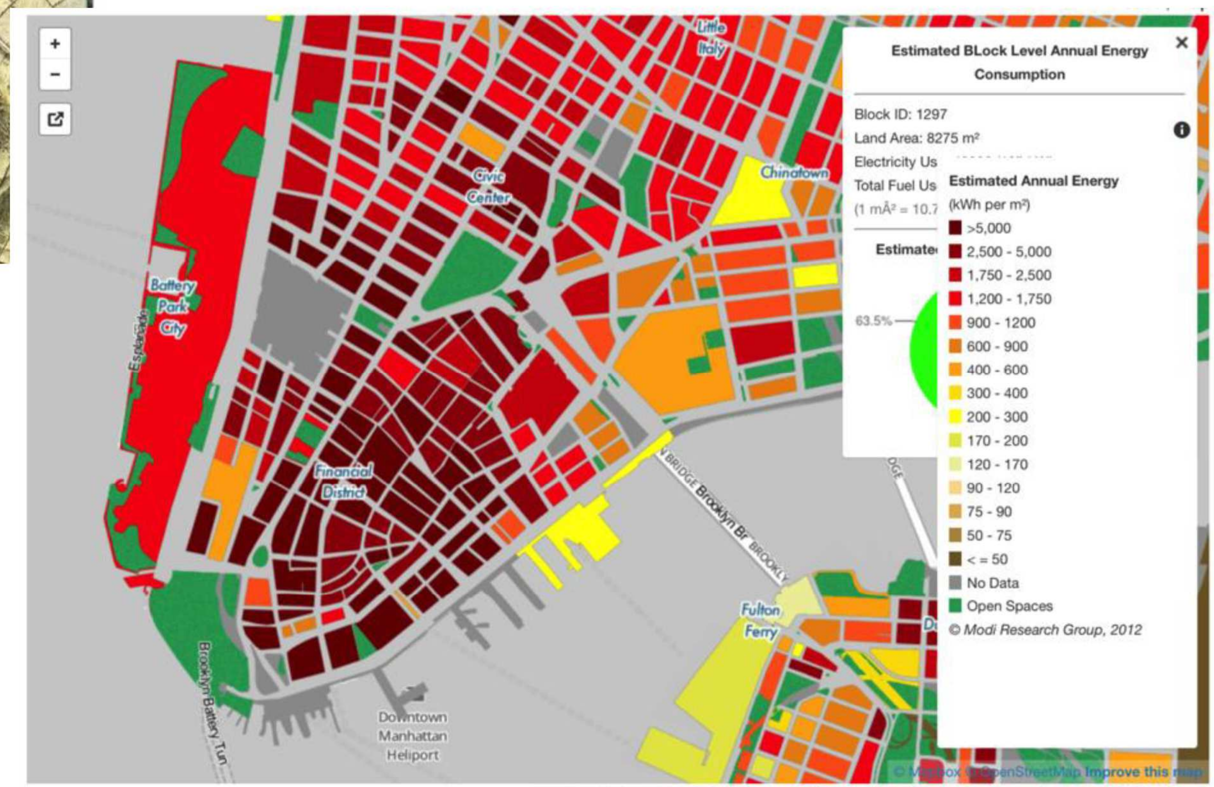
This map and the following

Data Source: [Spatial distribution of urban building energy consumption by end use   B. Howard, L. Parshall, J. Thompson, S. Hammer, J. Dickinson, V. Modi]



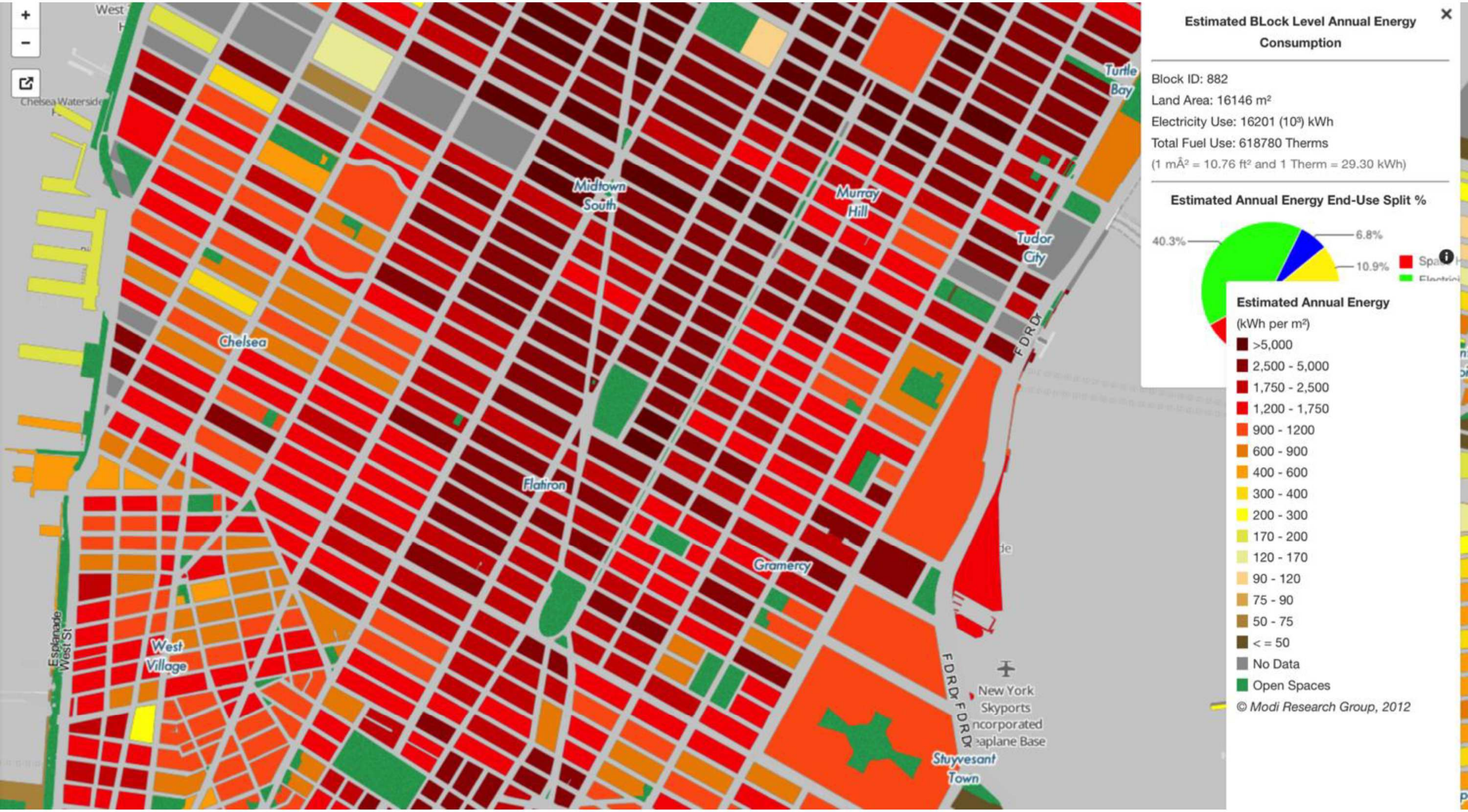
From New Amsterdam to Manhattan Financial district





Data Source: Spatial distribution of urban building energy consumption by end use
B. Howard, L. Parshall, J. Thompson, S. Hammer, J. Dickinson, V. Modi

Around Madison square

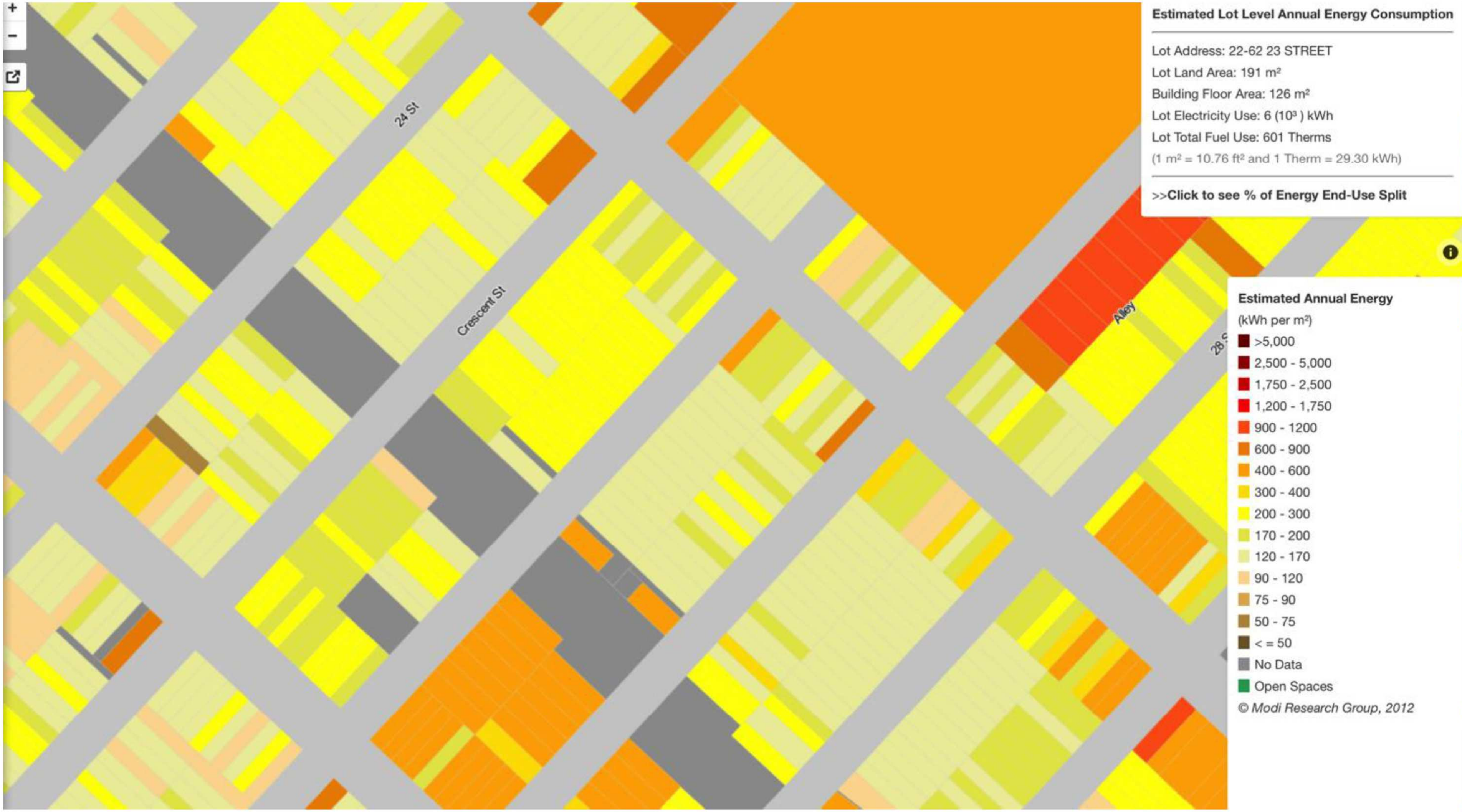


Manhattan Madison square area

The seemingly homogeneous landscape at block scale reveals high heterogeneity at plot scale

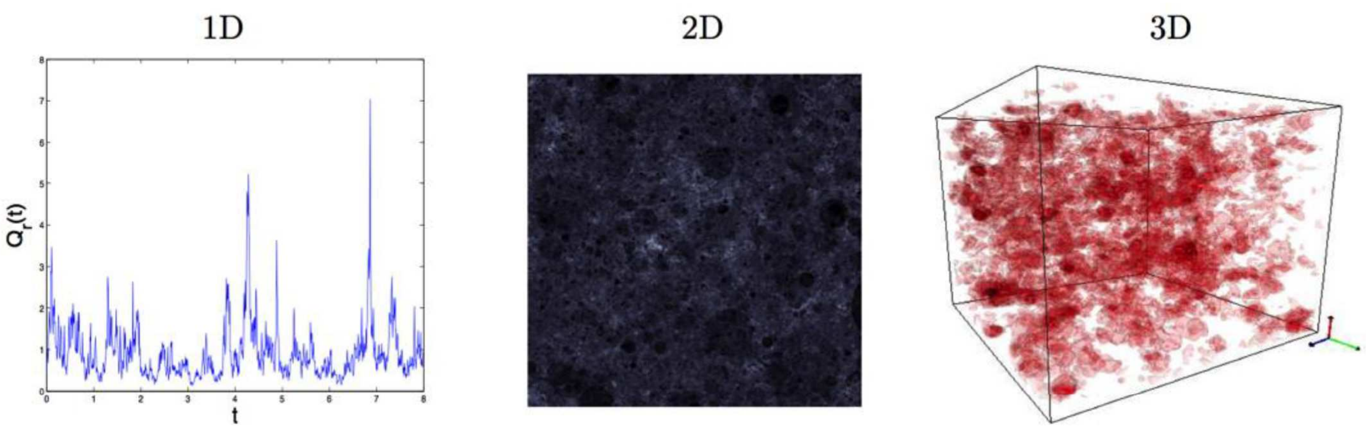

Estimated BLock Level Annual Energy Consumption
Block ID: 882
Land Area: 16146 m²
Electricity Use: 16201 (10³) kWh
Total Fuel Use: 618780 Therms
(1 mÂ² = 10.76 ft² and 1 Therm = 29.30 kWh)
Estimated Annual Energy End-Use Split %
40.3%
6.8%
10.9%
Estimated Annual Energy
(kWh per m²)
>5,000
2,500 - 5,000
1,750 - 2,500
1,200 - 1,750
900 - 1200
600 - 900
400 - 600
300 - 400
200 - 300
170 - 200
120 - 170
90 - 120
75 - 90
50 - 75
< = 50
No Data
Open Spaces
© Modi Research Group, 2012
E 28 St
E 27 St
E 26 St
E 25 St
E 24 St

More residential Brooklyn shows less variations in energy intensity.
The fractal landscape is less bumpy



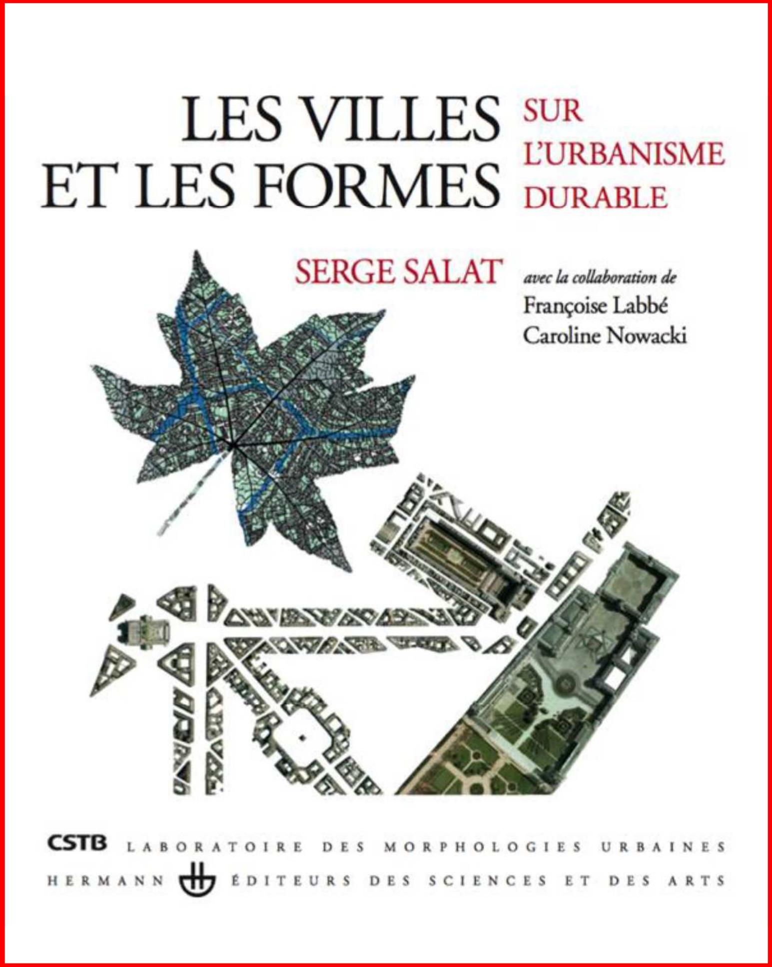
24 St
Crescent St
Alley
28 S
Estimated Lot Level Annual Energy Consumption
Lot Address: 22-62 23 STREET
Lot Land Area: 191 m²
Building Floor Area: 126 m²
Lot Electricity Use: 6 (10³) kWh
Lot Total Fuel Use: 601 Therms
(1 m² = 10.76 ft² and 1 Therm = 29.30 kWh)
>>Click to see % of Energy End-Use Split
Estimated Annual Energy
(kWh per m²)
>5,000
2,500 - 5,000
1,750 - 2,500
1,200 - 1,750
900 - 1200
600 - 900
400 - 600
300 - 400
200 - 300
170 - 200
120 - 170
90 - 120
75 - 90
50 - 75
< = 50
No Data
Open Spaces
© Modi Research Group, 2012

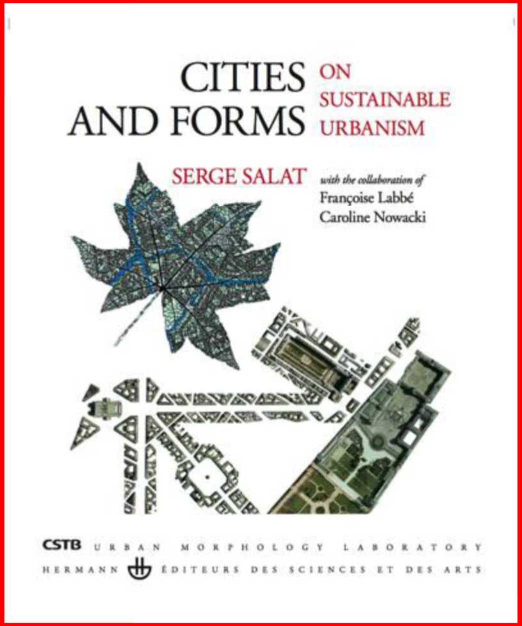
The multiscale, multi-parameters, multifractal structure of cities requires a multifractal description in a scalar field of N dimensions

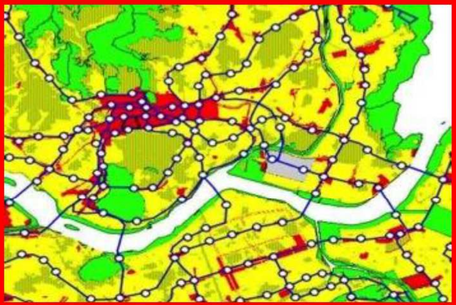


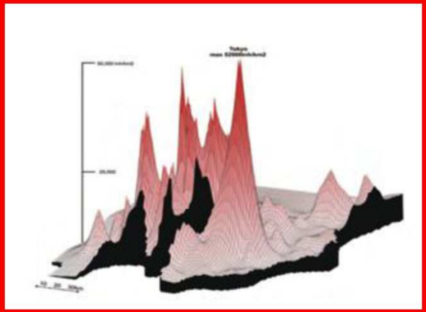
Exemples de cascades infiniment divisibles respectivement en dimensions 1, 2 et 3.

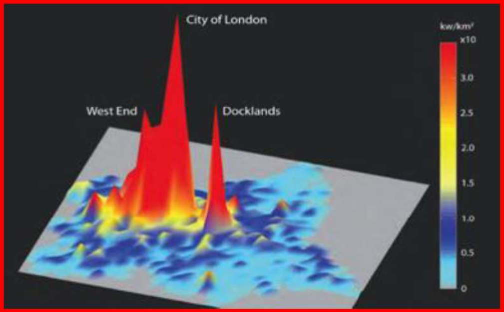
More on Multifractal Urban Forms



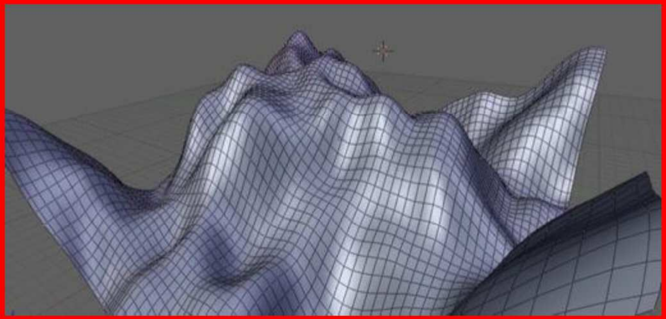








Thank you for your attention



serge.salat@free.fr

www.urbanmorphologyinstitute.org